

**Problem 1.** *There is no rational number whose square is 2.*

*Proof.* Assume, for contradiction, that there exist integers  $p$  and  $q$  satisfying

$$\frac{p}{q} = \sqrt{2},$$

where  $p/q$  is a rational number in lowest terms. By squaring, this is the same as  $\frac{p^2}{q^2} = 2$ , and by clearing denominators it is the same as

$$p^2 = 2q^2.$$

□

**Problem 2.** (a) *The negation of “For all real numbers satisfying  $a < b$ , there exists  $n \in \mathbb{N}$  such that  $a + (1/n) < b$ ” is*

(b) *The negation of “There exists a real number  $x > 0$  such that  $x < 1/n$  for all  $n \in \mathbb{N}$ ” is*

(c) *The negation of “Between every two distinct real numbers there is a rational number” is*

**Problem 3.** *Suppose  $a$  and  $b$  are real numbers. Then*

(a)  $|a - b| \leq |a| + |b|$

(b)  $||a - b|| \leq |a - b|$

*Proof.*

□

**Problem 4.** *Give an example of each, or state that it is impossible.*

(a)  $f : \mathbb{N} \rightarrow \mathbb{N}$  that is one-to-one but not onto.

(b)  $f : \mathbb{N} \rightarrow \mathbb{N}$  that is onto but not one-to-one.

(d)  $f : \mathbb{N} \rightarrow \mathbb{Z}$  that is one-to-one and onto.

**Problem 5.** *There exists an infinite collection of sets  $A_1, A_2, A_3, \dots$  with the properties that every  $A_i$  has an infinite number of elements, and  $A_i \cap A_j = \emptyset$  for all  $i \neq j$ , and  $\bigcup_{i=1}^{\infty} A_i = \mathbb{N}$ .*

*Proof.*

□