Review Guide for In-Class Midterm Exam 1 on Friday, 3 October 2025

Midterm Exam 1 is *closed book* and *closed notes*. No technology is allowed. Please bring nothing but a writing implement.

The Exam will cover sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 2.2, and 2.3 of the textbook.[1] The questions on the Exam will be of these types: prove propositions, state definitions and axioms, state theorems, and give examples with certain properties. You are expected to use reasonable and common notation; feel free to define your terms for clarity.

This Review Guide list *specific material that might appear on the exam.* Material which is *significantly different* from what is listed below will *not* appear. My goal is to only include topics that have appeared on homework and in lecture, and closely related things. You might be asked to state and prove the squeeze theorem for sequences, for example. However, I will *not* ask you to "state theorem 1.4.3," or anything like that which would require remembering locations in the book. (Numbers are listed below for easy of locating.)

Strongly recommended: Get together with other students and work through this Review Guide. Be honest with yourself about what you can easily prove, versus what you should think harder about and/or practice. Talk it through and learn!

Definitions. Be able to state and use the definition:

- natural numbers $\mathbb{N}$, integers $\mathbb{Z}$, rational numbers $\mathbb{Q}$, irrational number
- *all definitions in section 1.2:* set union, set intersection, empty set, disjoint, set inclusion, set complement, function, domain of a function, range of a function, absolute value function, triangle inequality
- one-to-one function, onto function, bijection
- upper bound, lower bound, bounded above, bounded below (for real sets)
- $\sup A$, $\inf A$ for a real set A
- maximum, minimum for a real set
- a subset is dense in $\mathbb{R}$
- finite set, infinite set
- countable set, uncountable set
- $A \sim B$: the sets A, B have the same cardinality
- power set of a set
- algebraic real number (Exercise 1.5.9)
- real sequence
- a real sequence converges ($\lim_{n\to\infty} a_n = a$) or diverges
- ϵ-neighborhood of a
- bounded sequence

[1] S. Abbott, *Understanding Analysis*, 2nd edition, Springer Press 2015

Concepts. Of course, you will be required to prove things, including one-way implications ($P \implies Q$) and equivalences ($P \iff Q$). For the latter goal one generally proves each direction in turn, and you are expected to make it clear which direction is which. Similarly, you might be asked to show set inclusion ($A \subseteq B$) or set equality ($A = B$), and for the latter one often shows inclusion both ways.

In addition, be able to use these proof techniques when appropriate:

- negate a proposition including "for all" and "there exists" quantifiers
- proof by contradiction
- proof by mathematical induction

and be able to state

- the axiom of completeness

Theorems. Be able to use and prove[2] these theorems.

- there is no rational number whose square is 2 (Thm 1.1.1)
- triangle inequality (Example 1.2.5)
- two real numbers are equal if and only if . . . (Thm 1.2.6)
- if $f : \mathbb{R} \to \mathbb{R}$ is a function and A, B are real subsets then $f(A \cup B) = f(A) \cup f(B)$ and $f(A \cap B) \subseteq f(A) \cap f(B)$ (Exercise 1.2.7)
- equivalent statement of $s = \sup A$ using $\epsilon > 0$ (Lem 1.3.8)
- nested interval property (Thm 1.4.1)
- Archimedean properties (Thm 1.4.2)
- density of $\mathbb{Q}$ in $\mathbb{R}$ (Thm 1.4.3)
- there exists $\alpha \in \mathbb{R}$ so that $\alpha^2 = 2$ (Thm 1.4.5) ← **proof not expected**
- $\mathbb{Q}$ is countable (Thm 1.5.6(i))
- $\mathbb{R}$ is uncountable, *via nested interval* (Thm 1.5.6(ii)) ← **proof not expected**
- if $A \subseteq B$ and B is countable . . . (Thm 1.5.7)
- countable union of finite sets is countable
- finite union of countable sets is countable (Thm 1.5.8(i))
- countable union of countable sets is countable (Thm 1.5.8ii) ← **proof not expected**
- open interval $(0, 1)$ is uncountable (Thm 1.6.1)
- Cantor's theorem (Thm 1.6.2) ← **proof not expected**
- limits are unique (Thm 2.2.7)
- convergent sequences are bounded (Thm 2.3.2)
- algebraic limit theorem (Thm 2.3.3)
- order limit theorem (Thm 2.3.4)

Examples. Every numbered Example, in the sections we covered, is fair game, as are closely-related examples. Every question on the homework Assignments 1–4, of the form "provide an example (with these properties)," is fair game, as are closely-related examples.

[2]Except if I say **proof not expected**, of course. But please read and understand these proofs!