Final Exam: Prove 4 Theorems

Tuesday, 9 December, 1:00pm–3:00pm, Chapman 204

The in-class Final Exam is different from the Midterms, but it is of modest length and it has a clear path for preparation. You will prove 4 major theorems which were important during the semester. The specific theorems are listed below, and you do have some choices. I will grade your proofs for completeness, correctness, and clarity.

You will have this document in your hand when you do the Final Exam. However, you must understand and remember what you want to write, because **you may NOT bring any notes or books or electronics to the Exam.**

To prepare for the Final Exam, you are strongly encouraged to draft and practice your proofs. Please do these things during your preparation:

- read and understand the proofs in the textbook, and fill in any missing parts,
- draft your proofs,
- get feedback on your drafts from other students/friends/family/pets, or me, and
- decide in advance how you will remember enough details so that you can recreate the proofs during the Exam itself.

Directions. Prove theorems **A** and **B**. Then choose one theorem from each category **C** and **D**, and prove it. Put only one proof on each sheet of paper. Write "**A**" or "**C2**" etc. in the top left corner of the sheet, and write your name in the upper right. Start each proof with "Proof." and end it with " $\Box$."

A. Monotone Convergence Theorem (Thm 2.4.2) If a sequence is monotone and bounded then it converges.

B. Bolzano-Weierstrass Theorem (Thm 2.5.5) Every bounded sequence contains a convergent subsequence.

C1. Density of $\mathbb{Q}$ (Thm 1.4.3) For every two real numbers a and b with $a < b$, there exists a rational $r \in \mathbb{Q}$ satisfying $a < r < b$.

C2. $(0, 1)$ is Uncountable (Thm 1.6.1) The open interval $(0, 1) = \{x \in \mathbb{R} : 0 < x < r\}$ is uncountable.

D1. Heine-Borel Theorem (Thm 3.3.4) A set $K \subseteq \mathbb{R}$ is compact if and only if it is closed and bounded.

D2. Continuous Preserves Compactness (Thm 4.4.1) Let $f : A \to \mathbb{R}$ be continuous on A. If $K \subseteq A$ is compact then $f(K)$ is compact.

D3. Continuous Preserves Connectedness (Thm 4.5.2) Let $f : G \to \mathbb{R}$ be continuous on A. If $E \subseteq G$ is connected then $f(e)$ is connected.