

min $c^T x$ subject to $Ax = b$, $x \geq 0$ where

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad c = \begin{bmatrix} -2 \\ -4 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \{3, 4\}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad c_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \hat{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$N = \{1, 2\}, \quad N = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \quad c_N = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$\underline{B^T y = c_B} \implies y = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^T y} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$\hat{c}_N \geq 0$?: stop with optimum $\hat{c}_N \xrightarrow[\text{min}]{\text{index of}} t = \boxed{2} \rightarrow \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\hat{A}_t \leq 0$?: stop, unbounded $\left\{ \frac{\hat{b}_i}{\hat{a}_{i,t}} \right\} = \left\{ \frac{3}{1}, \frac{2}{1} \right\}$ $\xrightarrow[\text{min over } \hat{a}_{i,t} > 0]{\text{index of}} s = \boxed{4}$

$$B = \{3, 2\}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad c_B = \begin{bmatrix} 0 \\ -4 \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \hat{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$N = \{1, 4\}, \quad N = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \quad c_N = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$\underline{B^T y = c_B} \implies y = \begin{bmatrix} 0 \\ -4 \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^T y} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$$

$\hat{c}_N \geq 0$?: stop with optimum $\hat{c}_N \xrightarrow[\text{min}]{\text{index of}} t = \boxed{1} \rightarrow \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\hat{A}_t \leq 0$?: stop, unbounded $\left\{ \frac{\hat{b}_i}{\hat{a}_{i,t}} \right\} = \left\{ \frac{1}{3}, X \right\}$ $\xrightarrow[\text{min over } \hat{a}_{i,t} > 0]{\text{index of}} s = \boxed{3}$

$$B = \{1, 2\}, \quad B = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \quad c_B = \begin{bmatrix} -2 \\ -4 \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \hat{b} = \begin{bmatrix} \frac{1}{3} \\ \frac{7}{3} \end{bmatrix}$$

$$N = \{3, 4\}, \quad N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad c_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{1}{3} \\ \frac{7}{3} \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{B^T y = c_B} \implies y = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^T y} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$\hat{c}_N \geq 0$?: stop with optimum

index of
→
min

$$t = \square$$

$$\rightarrow \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$\hat{A}_t \leq 0$?: stop, unbounded

$$\left\{ \frac{\hat{b}_i}{\hat{a}_{i,t}} \right\} = \{$$

index of
→
min over $\hat{a}_{i,t} > 0$

$$s = \square$$

$$B = \{ \quad \}, \quad B = \begin{bmatrix} \\ \\ \end{bmatrix}, \quad c_B = \begin{bmatrix} \\ \\ \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \hat{b} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$N = \{ \quad \}, \quad N = \begin{bmatrix} \\ \\ \end{bmatrix}, \quad c_N = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$\underline{B^T y = c_B} \implies y = \begin{bmatrix} \\ \\ \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^T y} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$\hat{c}_N \geq 0$?: stop with optimum

index of
→
min

$$t = \square$$

$$\rightarrow \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$\hat{A}_t \leq 0$?: stop, unbounded

$$\left\{ \frac{\hat{b}_i}{\hat{a}_{i,t}} \right\} = \{$$

index of
→
min over $\hat{a}_{i,t} > 0$

$$s = \square$$