About your project

Goal. The goal of the Math 661 project is for you to focus on topics of particular interest, and become more familiar with certain optimization problems and algorithms than is possible with the brief coverage typical of the rest of the course.

Expectations. All projects must include at least one specific optimization problem and at least one specific optimization algorithm. You will implement at least one algorithm, i.e. by a code you write in MATLAB/PYTHON/JULIA/etc., and then apply your code to at least one example problem, to solve it.

However, your project may be application-driven (*choose the problems first*) or algorithm-driven (*choose the algorithms first*). See the flowchart on page 3.

Both mathematical analysis and numerical computation are required. Your analysis should use theory from the textbook[1] and/or from other references. Analysis is important as it shows you have absorbed ideas from the course, and because it distinquishes between algorithms. Once your code is running you should provide some empirical (numerical experimentation) evidence regarding the error and/or performance of your algorithm(s). Numerical evidence shows that you understood the algorithm well enough to implement it correctly.

The problem(s) you choose must be in the following form:

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{subject to} \quad \begin{array}{ll} g_i(x) = 0, & i \in \mathcal{E}, \\ g_i(x) \ge 0, & i \in \mathcal{I}, \end{array} \tag{1}$$

(Of course you may replace $\min$ with $\max$.) If your project is algorithm-driven then you must identify which such problems are solved by your algorithms(s).

Form (1) describes a very large class of problems. Your chosen problems must be finite-dimensional, but they may arise from an infinite-dimensional source. The problems must be well-enough understood to allow you to both precisely identify the objective function $f(x)$ and to precisely identify a feasible set S defined by finitely-many equality and inequality constraints $g_i(x)$. It is o.k. if there are no constraints, with $\mathcal{E}$ and $\mathcal{I}$ as empty sets, but I may provide feedback on your Part I proposal which suggests you consider a constrained form of your problem.

Due dates. There are two due dates for the project:

Part I = project proposal: **Part I is due Friday 11 November at the start of class.**

There are no format requirements for the proposal, but it must be **two pages or less**. It should precisely say what problem(s) or algorithm(s) you will address. If application-driven it should explain *briefly* where the optimization problem(s) came from. In any case it should briefly motivate your proposed choice(s) of algorithm(s). Several quality references are *expected*; online references are o.k. though many informal online documents are of low quality. Please make specific references to our

[1] Griva, Nash, and Sofer, *Linear and Nonlinear Optimization*, 2nd ed., SIAM Press 2009.

textbook when that is appropriate. Your proposal should talk though what the complete project will contain, to the degree possible.

Spending at least a few hours on thinking and research at this stage can be very effective, but I suggest that you spend at most 6 hours on Part I.

Part II = actual project: **The completed project is due Monday 12 December at 5pm.**

It should have the format as shown below on page 4. Please use the indicated section headings! The total length **must be 20 pages or less**; I will not accept longer projects. The total time spent on the whole project should be at most 25 hours.

The format expectations can be met by using the LaTeX template posted online at bueler.github.io/opt/projects.html, but this is certainly not required.

Choosing a topic. I will help you choose problem(s) and algorithm(s) of reasonable difficulty. The bigger the scope the easier it is to get lost in the application, or in difficulties with programming/debugging/analysis. Your Part I proposal allows me to give good feedback on the topic, a gentle nudge in the direction of a variation, or a different analysis to consider, or that you bite off less, and etc.

You may **not** choose a topic which will be adequately covered in lecture. For example, neither the basic simplex method nor basic line search is a good topic. However, for example, implementations of the simplex method which respect sparsity would be a great choice (Chapter 7). Comparing line search methods beyond backtracking, or quasi-Newton methods beyond BFGS, or trust-region methods, would all be good choices (Chapters 11 and 12). There are many constrained optimization algorithms we will not get to, especially Chapters 8, 10, and 16.

Here are three approaches to choosing a topic if you don't already have one:

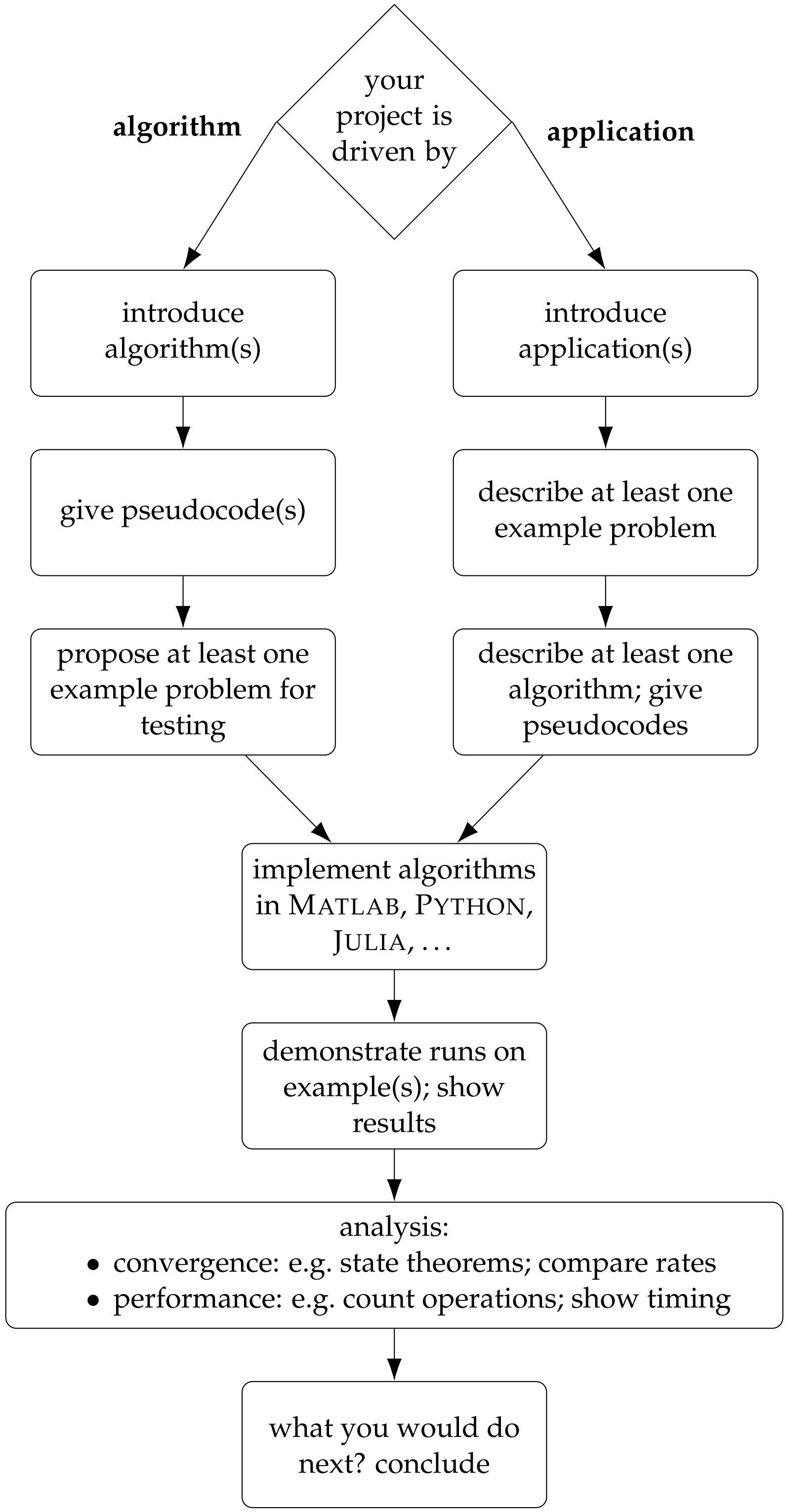
Approach 1: Inspiration from the Wikipedia page on mathematical optimization. See the "Major subfields," "Computational . . . techniques", and "Applications" sections.

en.wikipedia.org/wiki/Mathematical_optimization

Approach 2: Investigate skipped material from the textbook. Consider section(s) that you find interesting and which we did not cover.

Approach 3: A topic related to your thesis (if you have one). Please talk to your thesis advisor. It is reasonable to ask "are there optimization problems related to my expected thesis"? There may be significant algorithms which arise in your field of interest, or problems like parameter fitting, inverse modeling, or optimal design. There may be a paper to read about optimization in your field. Please **do not** cover territory comparable to your whole thesis; instead extract a little part, or extend a little part, and do it carefully. In any case, please make an effort to explain the context of your problem in your proposal.

Structure of the project. Here is a rough flow-chart. It aligns well with the section headings on the next page.



YOUR TITLE HERE

YOUR NAME

1. Introduction

PUT CONTENT HERE; PERHAPS CITE SOMETHING [1]

2. Algorithms [or Examples]

MORE CONTENT

3. Examples [or Algorithms]

CONTENT

4. Implementation

CONTENT

```
% MYCODE  This is my matlab implementation
x = 1:10;
y = randn(size(x));
plot(x,y)
z = 2+2
```

MORE CONTENT

```
>> mycode              % here I am running the code
z = 4
```

5. Results

CONTENT

6. Analysis

CONTENT; CITE SOMETHING? [2]

7. Conclusion

CONTENT

References

[1] A. Einstein (1905). *Zur Elektrodynamik bewegter Körper*, Annalen der Physik, 322 (10), 891–921.
[2] I. Griva, S. Nash, & A. Sofer (2009). *Linear and Nonlinear Optimization*, 2nd ed., SIAM Press.

Date: August 25, 2022.