Worksheet: If IEEE 754 had a 12-bit standard ...

The actual IEEE 754 standards for 64-bit double precision representation is cumbersome to play with, so for convenience we pretend here that the standard has a 12-bit version. It might look like this:

These 12 bits are organized to represent a *nonzero* number:

$$x = (-1)^s (1.b_1b_2b_3b_4b_5b_6b_7)_2 2^{(e_1e_2e_3e_4)_2 - (0111)_2}$$

Note that $(1.b_1b_2b_3b_4b_5b_6b_7)_2$ is called the *mantissa*. The power on the big 2 is the *exponent*. The special offset $(0111)_2$, equal to 7 in base ten, is called the *exponent bias*. We also define some exceptional cases:

- \bullet exponent bits $(0000)_2$ define the number zero or subnormal numbers
- exponent bits $(1111)_2$ define the other exceptions: $\pm \infty$ and NaN

(No further details of the $(1111)_2$ exceptions will be considered here.)

- (a) What is the largest real number that this system can represent? Show the bits.
- **(b)** What is the smallest positive number that this system can represent? (What is the first normal number to the right of zero?) Show the bits.
- (c) If we define $\epsilon_{\text{machine}}$ as the gap between 1 and the next representable number greater than 1, what is the value of $\epsilon_{\text{machine}}$ in this system?
- (d) What is the representation of zero? (Of +0? Of -0?) Show the bits.



(e)	What is the representation of 4? Show the bits.																
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(g)	In the interv	val [4,	, 8),	how	/ ma	ny n	uml	ers	can	be r	epre	sent	ed?				
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(h)	What is the	close	est n	umb	oer t	ο π =	= (3.	1415	59 · ·	•)103	? Sho	ow t	he bi	its.			
(i)	Exactly how	man	w d	lictir	act n	on-0	V COI	ation	aal r	num	hore	can	ho r	onr	ocont	tod i	n thi
	em? (<i>Include</i>)																
ехро	nent $(1111)_2$,	i.e. ±0	∞ ar	nd N	JaN.))								·	·		
(*)	C1 (1 1 1)			1		1	1										
(j)	Show the bit	ts of c	one :	subi	norn	nai n	umb	oer.				T	1]			
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