MATH 426 Numerical Analysis

Wed. 18 Sept.

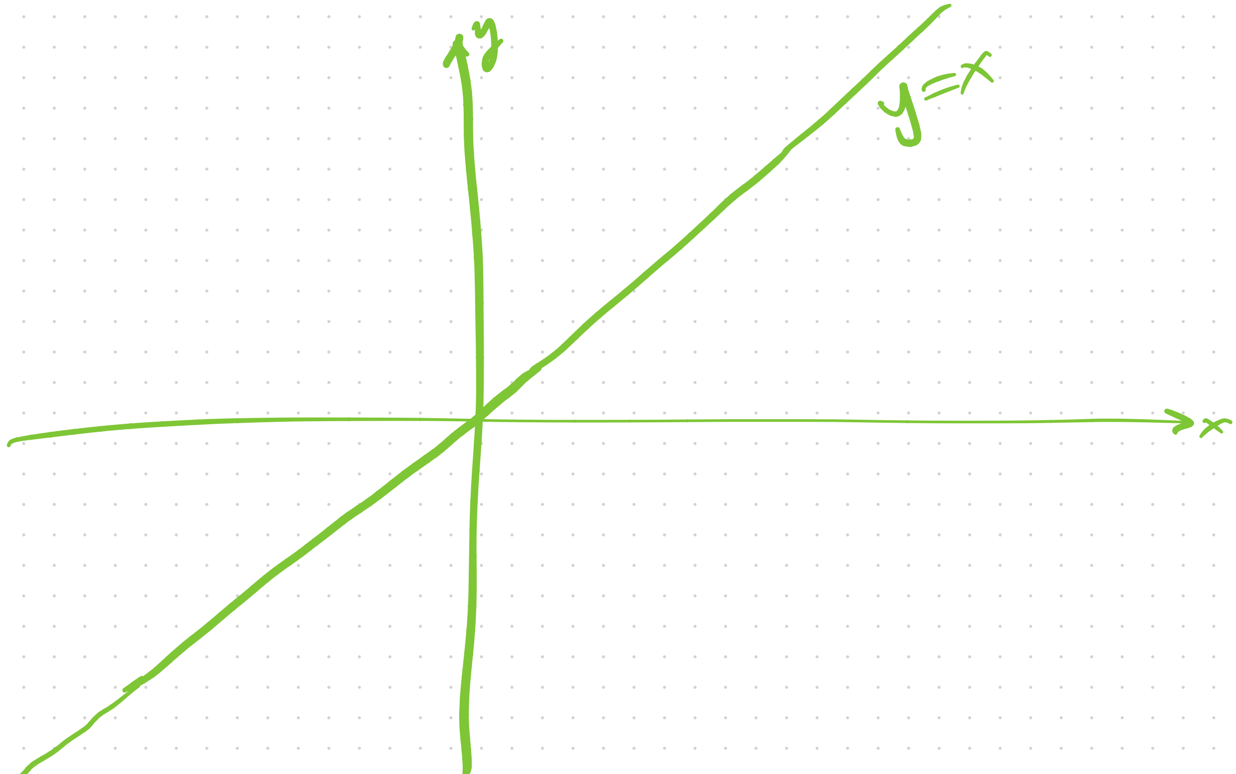
- Assignment #3 due <u>Friday</u>

<u>today</u>:
- fixed points (section 4.5)
- how your computer represents real numbers (Chapter 5)

Questions about Assignment 3?

#2 use axis([-2 2 -10 10]) or similar to show points and part of curve (polynomial); it is o.k. if the polynomial leaves the box

Section 4.5 fixed point methods



def: x_* is a fixed point of $\varphi(x)$ if $x_* = \varphi(x_*)$

Ex: (a) $x_{k+1} = \dfrac{x_k^2+6}{5}$

(b) $x_{k+1} = \dfrac{1}{2}\left(x_k + \dfrac{2}{x_k}\right)$

(c) $x_{k+1} = 5 - \dfrac{6}{x_k}$

do this at Matlab command line

for each of the above, start with $x_0 = 1$.
do the iterates $\{x_k\}$ converge? if so, how fast? also: graph all right-hand sides on common axes

Matlab:

```
>> x=1
>> x=(x^2+6)/5
⋮
```

repeat & see slow convergence to 2

```
>> x=1
>> x=0.5*(x+2/x)
⋮
```

see fast convergence to $\sqrt{2}$

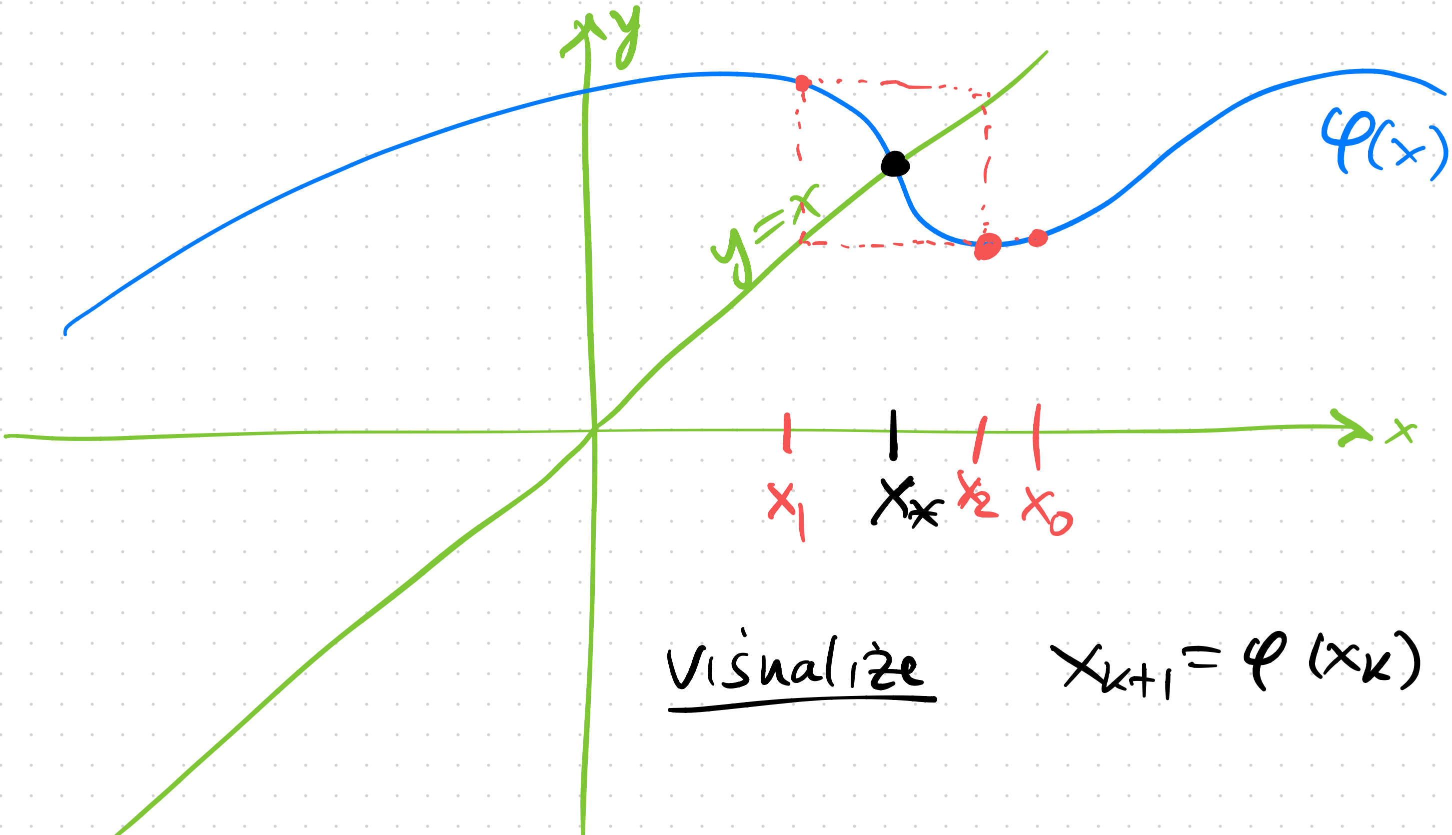
```
>> x=1
>> x=5-6/x
⋮
```

repeat and see (weird) and slow convergence to 3

Ex: (see #14 on p. 104)

put calculator in radians mode. pick a number at random. now repeatedly hit $\boxed{\cos}$ button. what happens? explain

general fixed point iteration picture



Visualize $x_{k+1} = \varphi(x_k)$

Theorem 4.5.1 If $\varphi(x)$ is C^1 and $|\varphi'(x)|<1$, and if x_* is a fixed point of $\varphi(x)$ then the fixed point iteration

$$x_{k+1} = \varphi(x_k)$$

converges: $\lim_{k\to\infty} x_k = x_*$

proof: use Taylor with $n=0$ and basepoint x_*:

$$x_{k+1} = \varphi(x_k) = \varphi(x_*) + \varphi'(\xi)(x_k - x_*)$$

$$= x_* + \varphi'(\xi)(x_k - x_*)$$

x_* is fixed point

<u>so</u>:

$$x_{k+1} - x_* = \varphi'(\xi)(x_k - x_*)$$

$$e_{k+1} = \varphi'(\xi)\, e_k$$

recall
$e_k = x_k - x_*$
by definitin

so

$$|e_{k+1}| = |\varphi'(\xi)|\,|e_k|$$

$$< 1 \cdot |e_k|$$

so $e_k \to 0$ so $x_k \to x_*$. $\blacksquare$

- go back and edit theorem to match book...

Newton method as a fixed-point iteration

- recall what Newton solves:

- recall Newton iteration:

Claim: Newton's method is fast because

- something different ...

Ex: apply Newton's method to

$$z^5 + 1 = 0,$$

and start with a complex number for z_0.

Soln: