Assignment #4

Due Monday, 30 September 2024, at the start of class

This Assignment is based on Chapters 4 and 5 of the textbook,[1] Please read these Chapters! When you turn in homework problems, please put the problems in the order they appear below. Also, the following two expectations will always apply:

1. If you use MATLAB, or the language of your choice, then the commands that you used must be shown, along with the results.
2. Please strive to minimize use of paper: edit your result to remove extra space *but* keep a clear distinction between your m-files, your input commands, and the computed results and/or figures.

Do the following exercises from the textbook:

CHAPTER 4

- Exercise 12 on page 104.
- Exercise 14 on page 104.
- Exercise 15 on page 104.

CHAPTER 5

- Exercise 3 on page 120.
- Exercise 8 on page 120.

Do the following additional problems:

P2. By plotting, check that the graphs $y = e^{-x^2}$ and $y = x^2$ cross between $x = 0.7$ and $x = 0.8$. Use the secant method, and MATLAB or language of your choice, to find the crossing point to within 10^{-14} using $x_0 = 0.7$ and $x_1 = 0.75$. (*You may write a script, or work at the command line, but show sufficient inputs/outputs to illustrate your understanding.*) Also use Newton's method with $x_0 = 0.75$ to do the same job. How many iterations of the secant method are needed? Of Newton's method?

P3. Consider the problem of solving $f(x) = x^3 - 7x + 2 = 0$ on the interval $[0, 1]$. Confirm, by using the intermediate value theorem, that there is a solution on that interval. Write down Newton's method for this problem as a fixed-point iteration $x_{k+1} = \varphi(x_k)$. What is $\varphi(x)$ in this case? By computing $\varphi'(x)$, and plotting it on the interval $[0, 1]$, find an interval I so that if x_0 is in I then you can be sure that Newton's method will converge. (*Use Theorem 4.5.1 to do the last part.*)

[1] Greenbaum & Chartier, *Numerical Methods: Design, Analysis, and Computer Implementation of Algorithms*, Princeton University Press 2012).