Worksheet: Recalling and using matrix factorizations.

Coming just before Midterm Quiz 1, the goal of this worksheet is to recall *which matrix factorizations are known already*, and *how they are used to solve linear systems* and *how they are used to build orthogonal projections*.

1. Assume $A \in \mathbb{C}^{m\times m}$ is square and invertible. State the **full SVD** of A, including properties of the factors, and then give the additional three steps needed to solve a linear system $Ax = b$. Make notes on the cost of the steps.

2. Assume $A \in \mathbb{C}^{m\times m}$ is square, invertible, *and diagonalizable*. State a **diagonalization factorization** of A, including properties of the factors, and then give the additional three steps needed to solve a linear system $Ax = b$.

3. Assume $A \in \mathbb{C}^{m\times m}$ is square and invertible. State the **full QR factorization** of A, including properties of the factors, and then give the additional two steps needed to solve a linear system $Ax = b$. Make notes on the cost of the steps.

A. Assume $A \in \mathbb{C}^{m\times n}$, $m \geq n$, is full-rank. State the **reduced SVD** of A, including sizes and properties of the factors. Then give a formula for the projection onto the range of A.

B. Assume $A \in \mathbb{C}^{m\times n}$, $m \geq n$, is full-rank. State the **reduced QR factorization** of A, including sizes and properties of the factors. What is the cost of this factorization? Then give a formula for the projection onto the range of A.