Assignment #4

Due Friday 3 October, at the start of class

Please read Lectures 5, 6, 7, and 8 in the textbook *Numerical Linear Algebra*, SIAM Press 1997, by Trefethen and Bau.

DO THE FOLLOWING EXERCISES FROM THE TEXTBOOK:

- **Exercise 5.2**
- **Exercise 5.3**
- **Exercise 6.1**
- **Exercise 6.3** You may use the SVD of A.
- **Exercise 6.4**

DO THE FOLLOWING ADDITIONAL PROBLEMS.

P10. **(a)** Give an example of a projector which is not an orthogonal projector.

(b) Show that if P is a projector and λ is an eigenvalue of P then $\lambda = 0$ or $\lambda = 1$.

(c) Show that if a projector is invertible then it is the identity.

P11. Show that if $A \in \mathbb{C}^{m\times m}$ with $\operatorname{rank}(A) = r$ then

$$\frac{1}{\sqrt{r}}\|A\|_F \le \|A\|_2 \le \|A\|_F.$$

(*Hint.* You may use a theorem in Lecture 5.) Note that computing $\|A\|_2$ requires $O(m^3)$ work (i.e. number of arithmetic operations) while computing $\|A\|_F$ requires only $O(m^2)$ work. Are there classes of matrices for which the inequalities above allow us to know the approximate size of $\|A\|_2$ with only the lesser amount of work?