Name: ________________________________

Math 614 Numerical Linear Algebra (Bueler) Wednesday 11 October 2023

Midterm Quiz 1

In-class or proctored. No book, notes, electronics, calculator, internet access, or communication with other people. 100 points possible.
<u>65 minutes</u> maximum!

1. Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

(a) (*5 pts*) What are the eigenvalues of A? (Give some brief explanation, or show work.)

(b) (*10 pts*) What is $\|A\|_2$? (*Hint.* Start as though you are finding singular values?)

2. Suppose A is an invertible $m \times m$ matrix which has SVD $A = U\Sigma V^*$. Let σ_j denote the ordered singular values of A.

(a) (*7 pts*) Write A^{-1} in terms of U, Σ, and V, and simplify.

(b) (*8 pts*) From **(a)**, explain why $\|A^{-1}\|_2 = 1/\sigma_m$.

Extra Credit A. (*3 pts*) Continuing problem **2** above, construct an SVD of A^{-1} from the SVD of A. (*Hint.* More subtle. Be careful with the ordering required for SVD.)

3. (*10 pts*) Show that if Q is unitary and λ is an eigenvalue of Q then $|\lambda| = 1$. (*Hint.* If z is a complex number then $|z|^2 = \bar{z}z$ where $\bar{z}$ is the conjugate.)

4. (*10 pts*) Suppose $q \in \mathbb{C}^m$ is a unit vector. Show $F = I - 2qq^*$ is unitary.

5. **(a)** (*10 pts*) Let $A \in \mathbb{R}^{m \times n}$ and $x \in \mathbb{R}^n$. Write a pseudocode or MATLAB code for the standard algorithm which computes the matrix-vector product Ax.

(b) (*5 pts*) Exactly how many floating point operations occur in the above algorithm?

6. **(a)** (*7 pts*) Suppose $\hat{Q} \in \mathbb{C}^{m \times n}$ has orthonormal columns. The product $\hat{Q}^*\hat{Q}$ has a particularly simple form. State what the form is and prove it.

(b) (*8 pts*) Continuing with the same matrix $\hat{Q}$, show that $P = \hat{Q}\hat{Q}^*$ is an orthogonal projector.

7. (*10 pts*) By any by-hand method, compute a reduced QR decomposition of

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

8. (*5 pts*) Suppose $A \in \mathbb{C}^{m\times n}$ and that $\|\cdot\|$ is a norm on $\mathbb{C}^n$ and that $\|\cdot\|'$ is a norm on $\mathbb{C}^m$. Define the *induced matrix norm* $\|A\|$. (*Full credit requires using the correct norms in the correct places.*)

9. (*5 pts*) Suppose P is an orthogonal projector. Show $I - P$ is an orthogonal projector.

Extra Credit B. (*3 pts*) Suppose $A \in \mathbb{R}^{3\times 3}$, and suppose we know the value of the following norms: $\|A\|_2 = 10$, $\|A^{-1}\|_2 = 1$, $\|A\|_F = 11$. Find all singular values of A.

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