

1st-versus-2nd order equations, and singular perturbations

1. Solve by hand:

$$u'(x) = 0, \quad u(0) = \alpha$$

2. Solve by hand:

$$u'(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

3. Solve by hand:

$$u''(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

4. Solve by hand:

$$0.01u''(x) + u'(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

5. Sketch the graphs of all solutions from the previous page on the same axes, in the case where $\alpha = 2$ and $\beta = -1$. (Make it big and label it clearly. Note that one of your sketches represents what happens for $\epsilon u''(x) + u'(x) = 0$, $u(0) = \alpha$, $u(1) = \beta$, when $\epsilon > 0$ is small.)

6. Sketch what you think the solution of

$$\epsilon u''(x) - u'(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

will look like if $\epsilon > 0$ is very small.