

1st-versus-2nd order equations, and singular perturbations

1. Solve by hand:

$$u'(x) = 0, \quad u(0) = \alpha$$

2. Solve by hand:

$$u'(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

3. Solve by hand:

$$u''(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

4. Solve by hand:

$$0.1u''(x) + u'(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

5. Sketch the graphs of all solutions from the previous page on the same axes, in the case where $\alpha = 2$ and $\beta = -1$. (*Make it big and label it clearly.*) Also sketch what happens in problem 4 if "0.1" is replaced by a much smaller $\epsilon > 0$; the ODE in question is $\epsilon u''(x) + u'(x) = 0$.

6. Sketch what you think the solution of

$$\epsilon u''(x) - u'(x) = 0, \quad u(0) = \alpha, \quad u(1) = \beta$$

will look like if $\epsilon > 0$ is very small.