Final Exam: 2 Short Case Summaries

Thursday, 4 May 2023, 10:15am–12:15pm, Brooks 302

The in-class Final Exam will be a bit different, but short. I would like you to summarize finite difference (FD) numerical analyses of two of the DE problems we have seen during the semester. That is, I would like you to write two *Case Summaries*.

Please choose one ODE and one PDE problem from the lists at the bottom. For each one, write a Case Summary on a separate sheet.

You may not bring notes to the final. However, you are **strongly encouraged to draft and practice your planned Case Summaries.** Make sure to read relevant sections of the textbook for your topics. Feel free to get feedback on your drafts from other students or faculty/friends/family/pets or me. Then think through, in preparation, how you will remember enough detail so as to recreate the Summary during the Final Exam itself.

Each summary should be at most a page. Equations are important, plus a few (at most three) sketches as needed, but please write complete sentences for the major ideas. A Summary which is *only* equations or sketches cannot achieve a good grade.

Except for special cases, a Summary should **state the problem** (DE and initial/boundary conditions), comment on or supply an **exact solution**, state one or two **FD discretization**(s), summarize the **truncation error**, and summarize **stability**. Then it should consider and fill-in any relevant additional ideas for the problem and the chosen method; see "Consider:" below for the specific problems. The goal is *not* to prove anything, but to accurately and precisely summarize what we know—or we easily could know—about the problem and the available method(s), especially in advance of writing a code.

For each Summary your goal is to write something like a neat 10 or 15 minute whiteboard presentation of how you would solve the problem numerically, as you might give to a hypothetical fellow graduate student who has not seen that particular problem but who has taken similar-level classes. Give an executive summary, and remember to point out the basic things!

ODE (1) A general linear 2nd-order ODE BVP. *eqn (2.64)*
Consider: how to solve the linear system.

ODE (2) The nonlinear 2nd-order ODE BVP for a pendulum. *eqn (2.77)*
Consider: Newton's method, how to solve the linear Newton steps.

ODE (3) A non-homogeneous, constant-coefficient linear ODE IVP system. *eqn (5.6)*
Consider: matrix exponentials, compare ≥ 2 schemes, absolute stability regions.

PDE (a) The Poisson problem PDE in 2D. *eqn (3.5)*
Consider: ordering the unknowns, how to solve the linear system.

PDE (b) The time-dependent heat equation PDE in 1D. *eqn (9.1)*
Consider: compare ≥ 2 schemes.

PDE (c) The time-dependent scalar advection equation PDE in 1D. *eqn (10.1)*
Consider: compare ≥ 2 schemes.