SOLUTIONS

Worksheet: getting comfortable with matrix indexing

All of these problems use "a_{ij}" for the entry in row i and column j of a matrix A.
Do these problems with a group, if possible!

I. Write down **the** 3 by 3 matrix A whose entries are given by

(a) $a_{ij} = $ minimum of i and j

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

(b) $a_{ij} = (-1)^{i+j}$

$$A = \begin{bmatrix} +1 & -1 & +1 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{bmatrix}$$

(c) $a_{ij} = i/j$

$$A = \begin{bmatrix} \frac{1}{1} & \frac{1}{2} & \frac{1}{3} \\ \frac{2}{1} & \frac{2}{2} & \frac{2}{3} \\ \frac{3}{1} & \frac{3}{2} & \frac{3}{3} \end{bmatrix}$$

II. What words would you use to describe each of these classes of matrices? Give **a** 3 by 3 example in each class. Which matrix belongs to all four classes?

(a) $a_{ij} = 0$ if $i \neq j$

lower triangular matrices

$$A = \begin{bmatrix} 5 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & 9 & 1 \end{bmatrix}$$

(b) $a_{ij} = 0$ if $i < j$

diagonal matrices

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) $a_{ij} = a_{ji}$

symmetric matrices

$$A = \begin{bmatrix} -1 & 7 & 3 \\ 7 & -2 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(d) $a_{ij} = a_{1j}$

"same rows"?
↑
not a standard name

$$A = \begin{bmatrix} 5 & 3 & 7 \\ 5 & 3 & 7 \\ 5 & 3 & 7 \end{bmatrix}$$

Swap these answers!

separ?

only the zero matrix, because diagonal & same rows ⟹ all entries 0

III. Write a sum in terms of entries a_{ij} and b_{ij}. **Don't** look-up the formula!

(a) Suppose A is $m \times n$ and B is $n \times p$. Let $C = AB$. Write a formula for each entry of C:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

(b) Suppose A is $m \times 1$ and B is $1 \times p$, so A is actually a column vector and B is actually a row vector. Again let $C = AB$, and write a formula for c_{ij}. (This time it is very simple! You are writing-out how the *outer product* works.)

$$c_{ij} = a_{i1} b_{1j}$$

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