## Worksheet: Using the row-reduced echelon form

For each linear system $A \mathbf{x}=\mathbf{b}$ below I applied Matlab's rref () command to the augmented matrix $[A \mathbf{b}]$ to get the row-reduced echelon form $[R \mathbf{d}]$. Interpret it to answer the following questions:

- what is the rank of $A$ ?
- find special solutions which span the nullspace $N(A)$
- identify vectors which span the column space $C(A)$
- write down the general solution to the system $A \mathbf{x}=\mathbf{b}$

1. 

$$
\begin{aligned}
8 x_{1}+x_{2}+15 x_{3} & =-22 \\
3 x_{1}+5 x_{2}+x_{3} & =1 \\
4 x_{1}+9 x_{2}-x_{3} & =6
\end{aligned}
$$

$$
[R \quad \mathbf{d}]=\left[\begin{array}{cccc}
1 & 0 & 2 & -3 \\
0 & 1 & -1 & 2 \\
0 & 0 & 0 & 0
\end{array}\right]
$$

2. 

$$
\begin{aligned}
12 x_{1}-10 x_{2}+5 x_{3} & =-6 \\
-9 x_{1}-x_{2}-5 x_{3} & =-32 \\
x_{1}+3 x_{2}+12 x_{3} & =38
\end{aligned} \quad \Longrightarrow \quad[R \mathbf{d}]=\left[\begin{array}{cccc}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 4 \\
0 & 0 & 1 & 2
\end{array}\right]
$$

3. 

$$
\begin{aligned}
2 x_{1}-x_{2}+5 x_{3}+2 x_{4} & =5 \\
2 x_{1}+x_{2}-x_{3}+6 x_{4} & =7
\end{aligned} \quad \Longrightarrow \quad[R \mathrm{~d}]=\left[\begin{array}{ccccc}
1 & 0 & 1 & 2 & 3 \\
0 & 1 & -3 & 2 & 1
\end{array}\right]
$$

4. 

$$
\begin{aligned}
2 x_{1}+2 x_{2} & =4 \\
-x_{1}+x_{2} & =4 \\
5 x_{1}-x_{2} & =-8 \\
2 x_{1}+6 x_{2} & =16
\end{aligned}
$$

$$
[R \quad \mathbf{d}]=\left[\begin{array}{ccc}
1 & 0 & -1 \\
0 & 1 & 3 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right]
$$

5. In problem 4 there are four equations in two unknowns. In typical cases there would be no solutions at all. Show a representative $[R \mathbf{d}]$ when there are no solutions.
