## Worksheet: Using the row-reduced echelon form

For each linear system  $A\mathbf{x} = \mathbf{b}$  below I applied Matlab's rref() command to the augmented matrix  $[A \mathbf{b}]$  to get the row-reduced echelon form  $[R \mathbf{d}]$ . Interpret it to answer the following questions:

- what is the **rank** of *A*?
- find special solutions which span the nullspace N(A)
- identify vectors which span the column space C(A)
- write down the general solution to the system  $A\mathbf{x} = \mathbf{b}$

## 1.

$8x_1 + x_2 + 15x_3 = -22$			Γ1	0	2	-3
$3x_1 + 5x_2 + x_3 = 1$	$\Rightarrow$	$[R \mathbf{d}] =$	0	1	-1	2
$4x_1 + 9x_2 - x_3 = 6$			0	0	0	0

2.

$12x_1 - 10x_2 + 5x_3 = -6$			1	0	0	2]
$-9x_1 - x_2 - 5x_3 = -32$	$\Rightarrow$	$[R \mathbf{d}] =$	0	1	0	4
$x_1 + 3x_2 + 12x_3 = 38$			0	0	1	2

$$2x_1 - x_2 + 5x_3 + 2x_4 = 5 \implies [R \mathbf{d}] = \begin{bmatrix} 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & -3 & 2 & 1 \end{bmatrix}$$

4.



5. In problem 4 there are four equations in two unknowns. In typical cases there would be no solutions at all. Show a representative [R d] when there are no solutions.