## Worksheet: elimination, via row operations and matrices

Do these calculations with a group, if possible.
A. Consider this system of three equations in three unknowns:

$$
\begin{aligned}
x_{1}+x_{2}+x_{3} & =4 \\
-8 x_{1}+4 x_{2} & =-8 \\
5 x_{1}+7 x_{2}-4 x_{3} & =-7
\end{aligned}
$$

Do elimination, as described in lecture and in section 2.2, followed by back-substitution, to solve this system. Make sure to follow the standard ordering of operations, even when it is tempting to do some ad hoc steps! Show your work in a reasonable way; documenting each step here will help in problem $\mathbf{B}$ on the other side. (Hint. The entries of $\mathbf{x}$ are small integers.)
B. Write the system in problem $\mathbf{A}$ (previous page) using matrices and vectors, as $A \mathbf{x}=\mathbf{b}$ :

$$
A=\left[\begin{array}{l}
]
\end{array}\right]
$$

Next, write down the three elimination matrices that correspond to the row operations you did. That is, fill-in these $3 \times 3$ matrices:
$E_{21}=[\square], \quad E_{31}=[\square$
Finally, multiply-out matrices, as follows. You should see that all the elimination steps on the previous page are reproduced here.

$$
\begin{aligned}
& E_{21} A=\left[\begin{array}{l} 
\\
E_{31}\left(E_{21} A\right)
\end{array}\right] \\
& E_{21} \mathbf{b}=[ \\
& E_{32}\left(E_{31}\left(E_{21} A\right)\right)=\left[\begin{array}{l} 
\\
\hline
\end{array}\right]
\end{aligned}
$$

C. Thus elimination converts the linear system to an upper triangular system:

$$
A \mathbf{x}=\mathbf{b} \quad \longrightarrow \quad U \mathbf{x}=\mathbf{c}
$$

What are $U$ and $\mathbf{c}$ ?:

$$
U=[\quad], \quad \mathbf{c}=[]
$$

Do back-substitution to (again) solve the system and find $\mathbf{x}$.

