Worksheet: 2 views of linear equations

Do these sketches and calculations with a group, if possible.

A. Here are three equations in two unknowns:

$$2x + 2y = 6$$
$$x - 3y = -1$$
$$4x + y = 0$$

We can also write this as $A\mathbf{x} = \mathbf{b}$ where A is a 3×2 matrix and $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$ are vectors.

(i) Sketch the "row picture": sketch each equation as a line in the (x, y) plane. Do they intersect?

(ii) Sketch the "column picture": sketch each column of *A*, and also b, in three-dimensional space. Will you be able to find a linear combination of the columns of *A* which gives b?

(iii) Continuing problem **A**, change one entry of the right side so that the linear system does have a solution, and find that solution.

B. (i) Consider the new linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & 0 \\ 3 & 2 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix},$$

and $\mathbf{x} = (x_1, x_2, x_3)$ is unknown (*for now*). Sketch the row picture, that is, sketch each of the three equations as a plane. Note it is easier to sketch each plane on separate axes; show three sketches.

(ii) What is the solution of the system in part (i)?