## Worksheet: 2 views of linear equations

Do these sketches and calculations with a group, if possible.
A. Here are three equations in two unknowns:

$$
\begin{aligned}
2 x+2 y & =6 \\
x-3 y & =-1 \\
4 x+y & =0
\end{aligned}
$$

We can also write this as $A \mathbf{x}=\mathbf{b}$ where $A$ is a $3 \times 2$ matrix and $\mathbf{x}=\left[\begin{array}{l}x \\ y\end{array}\right], \mathbf{b}=\left[\begin{array}{c}6 \\ -1 \\ 0\end{array}\right]$ are vectors.
(i) Sketch the "row picture": sketch each equation as a line in the $(x, y)$ plane. Do they intersect?
(ii) Sketch the "column picture": sketch each column of $A$, and also $\mathbf{b}$, in three-dimensional space. Will you be able to find a linear combination of the columns of $A$ which gives $\mathbf{b}$ ?
(iii) Continuing problem A, change one entry of the right side so that the linear system does have a solution, and find that solution.
B. (i) Consider the new linear system $A \mathbf{x}=\mathbf{b}$ where

$$
A=\left[\begin{array}{ccc}
1 & 0 & -1 \\
2 & 2 & 0 \\
3 & 2 & 1
\end{array}\right], \quad \mathbf{x}=\left[\begin{array}{l}
0 \\
4 \\
6
\end{array}\right],
$$

and $\mathbf{x}=\left(x_{1}, x_{2}, x_{3}\right)$ is unknown (for now). Sketch the row picture, that is, sketch each of the three equations as a plane. Note it is easier to sketch each plane on separate axes; show three sketches.
(ii) What is the solution of the system in part (i)?

