

# SOLUTIONS

Math 314 Linear Algebra (Bueler)

21 January 2022 Not to be turned in!

## Worksheet: 2 views of linear equations

Do these sketches and calculations with a group, if possible.

A. Here are three equations in two unknowns:

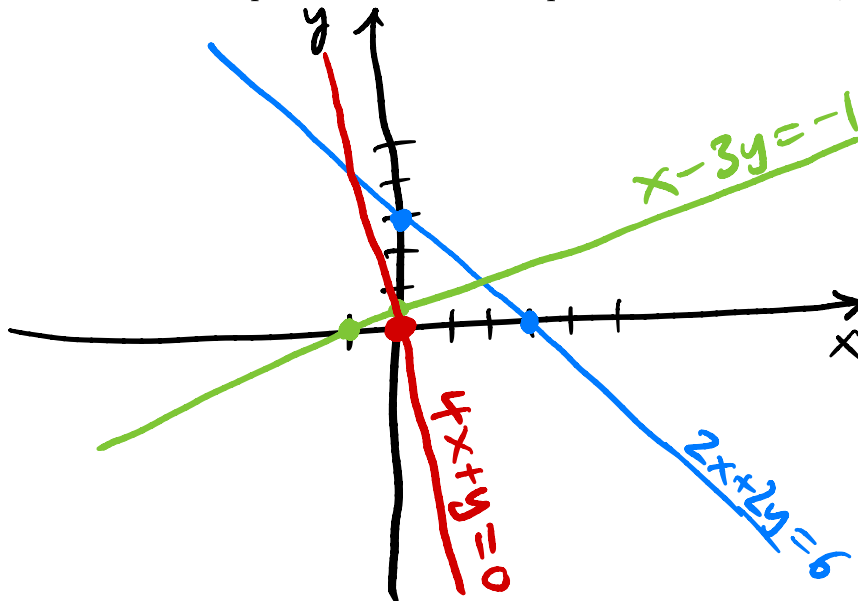
$$2x + 2y = 6$$

$$x - 3y = -1$$

$$4x + y = 0$$

We can also write this as  $A\mathbf{x} = \mathbf{b}$  where  $A$  is a  $3 \times 2$  matrix and  $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ ,  $\mathbf{b} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$  are vectors.

(i) Sketch the “row picture”: sketch each equation as a line in the  $(x, y)$  plane. Do they intersect?



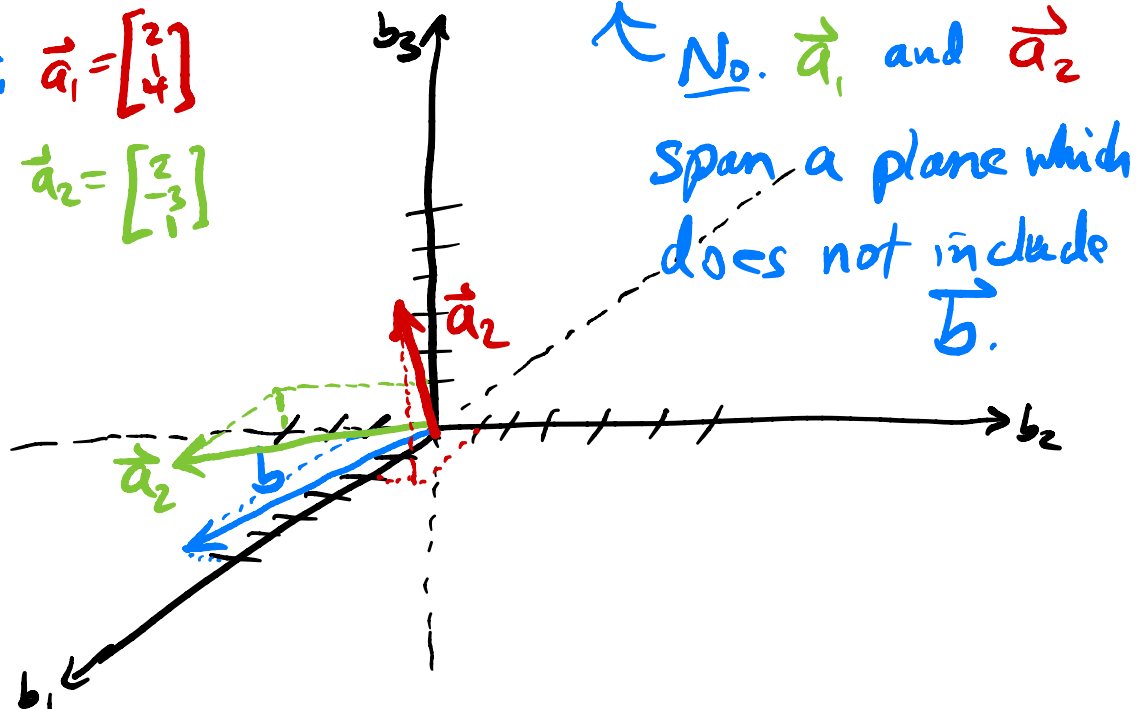
↑  
not in one  
location  
(which is what  
I meant to ask)

(ii) Sketch the “column picture”: sketch each column of  $A$ , and also  $\mathbf{b}$ , in three-dimensional space. Will you be able to find a linear combination of the columns of  $A$  which gives  $\mathbf{b}$ ?

$$A = \begin{bmatrix} 2 & 2 \\ 1 & -3 \\ 4 & 1 \end{bmatrix} \therefore \vec{a}_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\vec{a}_2 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$$



(iii) Continuing problem A, change one entry of the right side so that the linear system does have a solution, and find that solution.

method: pick two equations, solve those for  $(x, y)$ , then determine other entry of new  $\vec{b}$  by left side of other equation

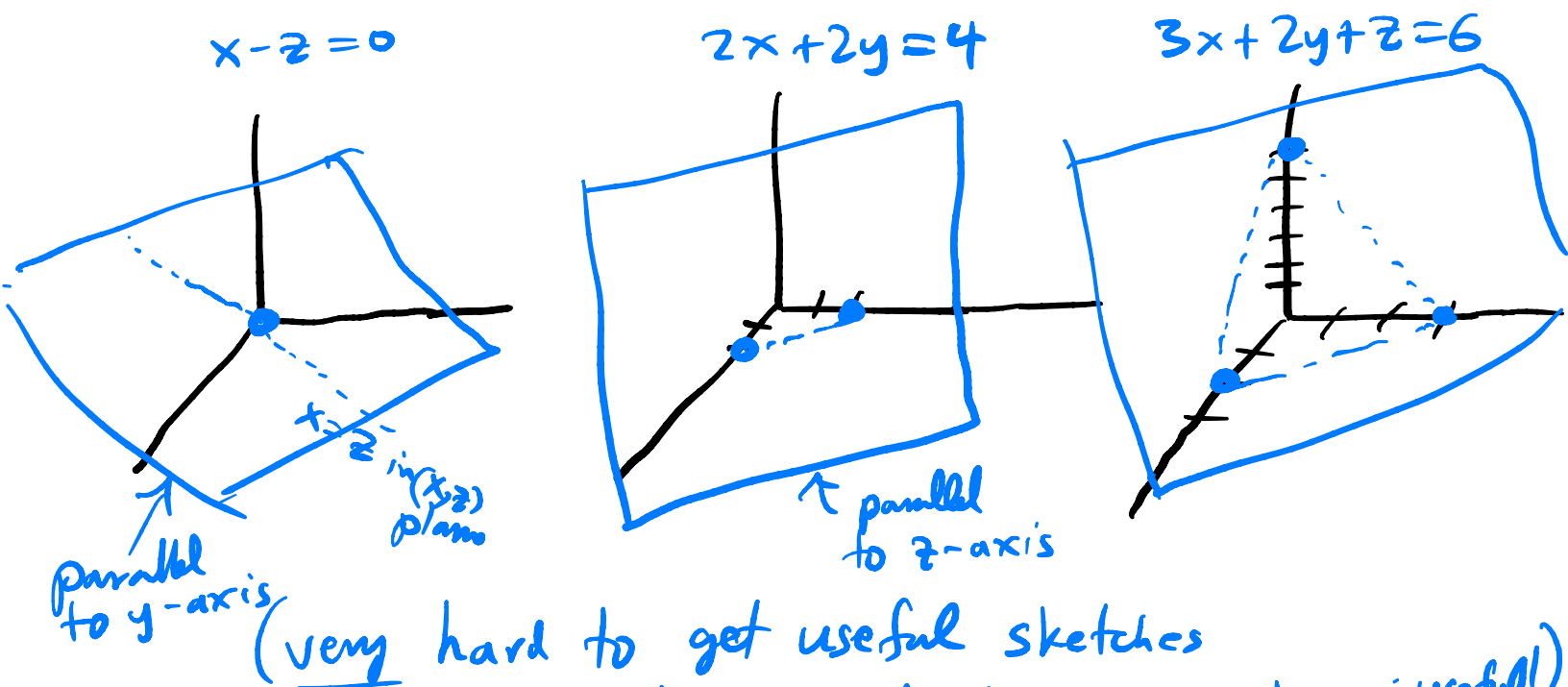
one example  $\left\{ \begin{array}{l} 2x + 2y = 6 \\ x - 3y = -1 \end{array} \right. \rightarrow \begin{array}{l} 2x + 2y = 6 \\ -4y = -4 \end{array} \rightarrow \begin{array}{l} y = 1 \\ x = 2 \end{array}$

new:  $\vec{b} = \begin{bmatrix} 6 \\ -1 \\ 9 \end{bmatrix}$  ← 3rd eq:  $4x + y = 9$

B. (i) Consider the new linear system  $Ax = b$  where

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & 0 \\ 3 & 2 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix},$$

and  $x = (x_1, x_2, x_3)$  is unknown (for now). Sketch the row picture, that is, sketch each of the three equations as a plane. Note it is easier to sketch each plane on separate axes; show three sketches.



(ii) What is the solution of the system in part (i)?

$$\begin{array}{l} x - z = 0 \\ 2x + 2y = 4 \\ 3x + 2y + z = 6 \end{array} \rightarrow \begin{array}{l} x - z = 0 \\ 2y + 2z = 4 \\ 2y + 4z = 6 \end{array} \rightarrow \begin{array}{l} x - z = 0 \\ 2y + 2z = 4 \\ z = 1 \end{array} \rightarrow \begin{array}{l} x = 1 \\ y = 1 \\ z = 1 \end{array}$$

(doing elimination) [back subst]