## Worksheet: Is it a subspace?

A vector space is a set of vectors with a defined addition operation and a scalar multiple operation. (Various sensible rules-p. 130—apply to those operations.) A subspace is a subset $S$ of the vector space for which any linear combination of elements from $S$ is in $S$.

You can verify that $S$ is a subspace by checking if 0 is in $S$, if $\mathbf{v}+\mathbf{w}$ is in $S$ whenever $\mathbf{v}$, $\mathbf{w}$ are in $S$, and finally if $c \mathbf{v}$ is in $S$ whenever $\mathbf{v}$ is in $S$ and $c$ is any real number.

For each problem below, sketch $S$ if possible, and otherwise describe it. Say whether $S$ is a subspace or not. If so, provide a brief justification. If not, describe an element that is not in $S$ but would be if $S$ were a subspace.

1. Vector space: $\mathbf{R}^{2}$. $S$ is the set of all points in the first quadrant of $\mathbf{R}^{2}$.
2. Vector space: all real-valued functions on the line. $S$ is the set of all polynomials.
3. Vector space: $\mathbf{R}^{3} . S$ is the set of all vectors $a \mathbf{i}+b \mathbf{j}+c \mathbf{k}$ where $a, b, c$ are integers.
4. Vector space: $\mathbf{R}^{2} . S=$ all solutions to $A \mathbf{x}=\mathbf{0}$ where $A=\left[\begin{array}{cc}2 & -3 \\ 6 & 7\end{array}\right]$.
5. Vector space: $\mathbf{R}^{2}$. $S=$ all solutions to $A \mathbf{x}=\mathbf{b}$ where $A=\left[\begin{array}{cc}2 & -3 \\ -6 & 9\end{array}\right]$ and $\mathbf{b}=\left[\begin{array}{c}-1 \\ 3\end{array}\right]$.
6. Vector space: $\mathbf{R}^{2} . S=$ all solutions to $A \mathbf{x}=\mathbf{0}$ where $A=\left[\begin{array}{cc}2 & -3 \\ -6 & 9\end{array}\right]$.
7. Vector space: $\mathbf{R}^{3}$. $S=$ all solutions to $A \mathbf{x}=\mathbf{0}$ where $A=\left[\begin{array}{ccc}3 & 2 & 1 \\ 0 & 1 & 1 \\ -6 & -3 & -1\end{array}\right]$.
