SOLUTIONS

Worksheet: Is it a subspace?

A *vector space* is a set of vectors with a defined addition operation and a scalar multiple operation. (Various sensible rules—p. 130—apply to those operations.) A *subspace* is a subset S of the vector space for which any linear combination of elements from S is in S.

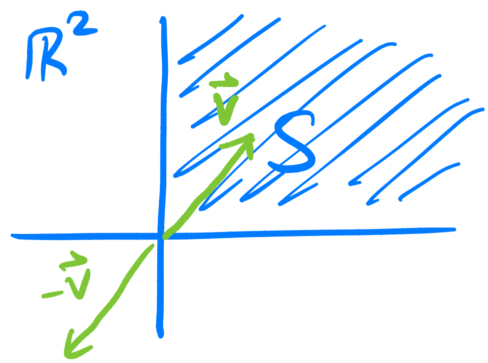
You can verify that S is a subspace by checking if 0 is in S, if $\mathbf{v}+\mathbf{w}$ is in S whenever $\mathbf{v}, \mathbf{w}$ are in S, and finally if $c\mathbf{v}$ is in S whenever $\mathbf{v}$ is in S and c is any real number.

For each problem below, sketch S if possible, and otherwise describe it. Say whether S is a subspace or not. If so, provide a brief justification. If not, describe an element that is not in S but would be if S were a subspace.

1. Vector space: $\mathbf{R}^2$. S is the set of all points in the first quadrant of $\mathbf{R}^2$.

no $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is in S

but $-\vec{v}$ is not



2. Vector space: all real-valued functions on the line. S is the set of all polynomials.

yes if $p(x)$ and $q(x)$ are in S then

$p(x) = a_0 + a_1 x + \dots + a_n x^n$ (degree n)

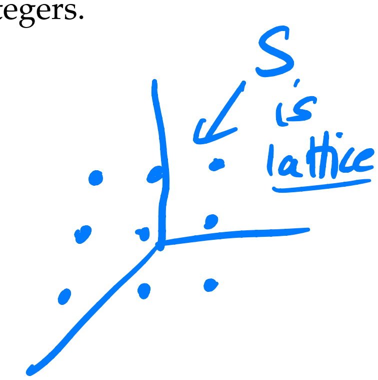
$q(x) = b_0 + b_1 x + \dots + b_m x^m$ (degree m)

then $c\,p(x)$ and $p(x)+q(x)$ are also polynomials of degree n and $\max\{n,m\}$ respectively

3. Vector space: $\mathbf{R}^3$. S is the set of all vectors $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ where a, b, c are integers.

no $\vec{v} = 1\cdot\hat{\imath} + 0\cdot\hat{\jmath} + 0\cdot\hat{k} = \hat{\imath}$ is in S

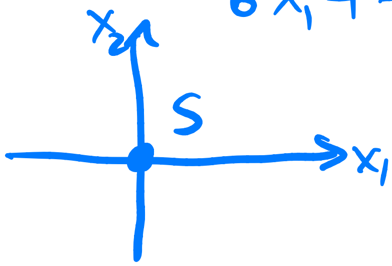
but $\frac{1}{2}\vec{v} = \frac{1}{2}\hat{\imath}$ is not in S



4. Vector space: $\mathbf{R}^2$. S = all solutions to $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 2 & -3 \\ 6 & 7 \end{bmatrix}$.

yes

$$\begin{aligned} 2x_1 - 3x_2 &= 0 \\ 6x_1 + 7x_2 &= 0 \end{aligned} \xrightarrow{\text{elim.}} \begin{aligned} 2x_1 - 3x_2 &= 0 \\ 16x_2 &= 0 \end{aligned} \Rightarrow \begin{aligned} x_1 &= 0 \\ x_2 &= 0 \end{aligned}$$

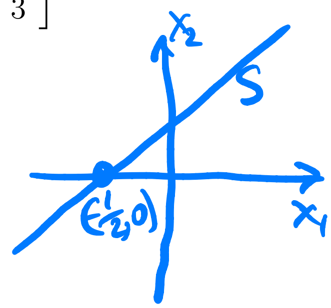


$S = \{\vec{0}\} = \left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\} = Z$ is the trivial vectorspace

5. Vector space: $\mathbf{R}^2$. S = all solutions to $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

no

$$\begin{aligned} 2x_1 - 3x_2 &= -1 \\ -6x_1 + 9x_2 &= 3 \end{aligned} \xrightarrow{\text{elim.}} \begin{aligned} 2x_1 - 3x_2 &= -1 \\ 0x_1 + 0x_2 &= 0 \end{aligned}$$



let $x_2 = t$. then $x_1 = \dfrac{-1+3t}{2} = -\frac{1}{2} + \frac{3}{2}t$

so $S = \left\{\begin{bmatrix} -1/2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3/2 \\ 1 \end{bmatrix}\right\}$ where t is any real number

so $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is not in S

6. Vector space: $\mathbf{R}^2$. S = all solutions to $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$.

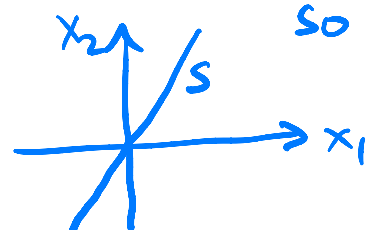
yes

$$\begin{aligned} 2x_1 - 3x_2 &= 0 \\ -6x_1 + 9x_2 &= 0 \end{aligned} \xrightarrow{\text{elim.}} \begin{aligned} 2x_1 - 3x_2 &= 0 \\ 0x_1 + 0x_2 &= 0 \end{aligned}$$

let $x_2 = t$

so $x_1 = \frac{3}{2}t$

so $S = \left\{ t\begin{bmatrix} 3/2 \\ 1 \end{bmatrix} \right\}$ for any t



7. Vector space: $\mathbf{R}^3$. S = all solutions to $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ -6 & -3 & -1 \end{bmatrix}$.

yes

$$\xrightarrow{\text{elim}} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$3x_1 + 2x_2 + x_3 = 0$
$x_2 + x_3 = 0$

$x_3 = t$
$x_2 = -t$
$x_1 = \frac{1}{3}(-t + 2t)$
$= \frac{1}{3}t$

$S = \left\{ t\begin{bmatrix} 1/3 \\ -1 \\ 1 \end{bmatrix} \right\}$ for any t