Worksheet: The four subspaces

For each matrix A below I show $R = \texttt{rref}(A)$, i.e. from Matlab. Answer the following questions:

- what are the dimensions of the four subspaces $C(A^\top)$, $C(A)$, $N(A)$, $N(A^\top)$?
- find a basis for each of the first three subspaces $C(A^\top)$, $C(A)$, $N(A)$

1.

$$A = \begin{bmatrix} 8 & 1 & 15 \\ 3 & 5 & 1 \\ 4 & 9 & -1 \end{bmatrix} \qquad \to \qquad R = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

2.

$$A = \begin{bmatrix} 12 & -10 & 5 \\ -9 & -1 & -5 \\ 1 & 3 & 12 \end{bmatrix} \qquad \to \qquad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.

$$A = \begin{bmatrix} 2 & -1 & 5 & 2 \\ 2 & 1 & -1 & 6 \end{bmatrix} \qquad \rightarrow \qquad R = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -3 & 2 \end{bmatrix}$$

4.

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \\ 5 & -1 \\ 2 & 6 \end{bmatrix} \qquad \rightarrow \qquad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5. For A in problem **4**, $\texttt{rref}(A^\top) = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -3 & 2 \end{bmatrix}$. From this, find a basis for $N(A^\top)$.