Math 314 Name :___________________

Exam 2 April 10, 2013

Show all your work.

1. (16 pts.) Let $A = \begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}$.

   (a) (6 pts.) Find the eigenvalues of A.

   (b) (6 pts.) For each eigenvalue, determine an eigenvector.

   (c) (4 pts.) Give matrices Λ and S for a diagonalization $A = S\Lambda S^{-1}$.

2. (11 pts.) In $\mathbb{R}^4$, a subspace V has basis $(1, 2, -1, 1)$, $(2, 0, 1, 1)$. Find a basis for $V^\perp$.

3. (15 pts.) The vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$

span a 3-d subspace of $\mathbb{R}^4$. Find an orthonormal basis for that subspace.

4. (15 pts.—5 pts. each) Suppose you wanted to fit a straight line $y = mx + b$ to the (x, y) data points

$$(-1, 2),\ (0, 1),\ (1, -2),\ (2, -2).$$

(a) Give, in matrix form, a system of 4 equations in 2 unknowns that you would *like* to solve to find this line, even though this system has no solution.

(b) Give, in matrix form, a system of 2 equations in 2 unknown that you could solve to find the least-squares best fit line. (Do NOT solve them.)

(c) The idea behind what is being done here is that if $A\mathbf{x} = \mathbf{b}$ has no solution due to suspected errors in $\mathbf{b}$, then we should replace $\mathbf{b}$ with a different vector so the system becomes solvable. This vector is found by projecting $\mathbf{b}$ onto what?

5. (12 pts.) Suppose

$$A = \begin{pmatrix} 2 & 1 & 1 & -1 & 2 \\ 4 & 2 & 3 & -1 & 5 \\ 2 & 1 & -2 & -4 & -1 \\ -2 & -1 & 0 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) (3 pts.) Give a basis for the rowspace of A.

(b) (3 pts.) Give a basis for the column space of A.

(c) (3 pts.) What is the dimension of the nullspace of A?

(d) (3 pts.) What is the dimension of the left nullspace of A?

6. (10 pts.) Calculate the determinant

$$\begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 3 & 3 \\ 2 & 1 & -1 & 1 \\ 1 & 1 & 2 & 4 \end{vmatrix}.$$

7. (21 pts.—3 pts. each) Fill in the blanks:

(a) If the columns of an $m \times r$ matrix A are independent, then a matrix to project $\mathbb{R}^m$ onto the column space of A can be found using the formula _________

(b) If the 'big formula' for the determinant of a 5×5 matrix were written out, it would be a sum of _________ terms, each of which is ± 1 times a product of _________ entries of the matrix.

(c) The inverse of an orthogonal matrix Q is most easily computed by _________

(d) If a 3×3 matrix A has eigenvalues $3, 1, -2$, then the eigenvalues of A^3 are _________.

(e) If $\det B = -13$, then $\det B^T =$ _________.

(f) If P is a 5×5 matrix that projects vectors in $\mathbb{R}^5$ onto a 3-dimensional subspace W, then the 5 eigenvalues of P will be _________.

(g) If $|A| = 2/5$, then $|A^{-1}| =$ _________.