Homework #7

Due Monday 14 March, 2022 at 11:59pm.

Submit as a single PDF via Gradescope, linked from the Canvas page
canvas.alaska.edu/courses/7017
Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions are linked at
bueler.github.io/math314/resources.html
P Problems will be graded for correctness. When grading these, I expect you to write explanations using complete sentences!

Put these Textbook Problems first on your PDF, in this order.

from Problem Set 3.3, pages 157–162: # 3, 4, 10, 16, 18, 21, 30

from Problem Set 3.4, pages 174–179: # 1, 2, 11, 13, 16, 32, 38

Put these ***P*** *Problems next on your PDF, in this order.*

P28. Include $\boldsymbol{b}$ as a fourth column in elimination on the following system, that is, use an augmented matrix. Identify a condition on b_1, b_2, b_3 so that this system is solvable:

$$\begin{aligned} x + 2y - 2z &= b_1 \\ 2x + 5y - 4z &= b_2 \\ -2x - 6y + 4z &= b_3 \end{aligned}$$

P29. Give examples of matrices A for which the number of solutions to $A\boldsymbol{x} = \boldsymbol{b}$ is

- **(a)** 0 or 1, depending on $\boldsymbol{b}$
- **(b)** ∞, regardless of $\boldsymbol{b}$
- **(c)** 0 or ∞, depending on $\boldsymbol{b}$
- **(d)** 1, regardless of $\boldsymbol{b}$

P30. **(a)** If $A\boldsymbol{x} = \boldsymbol{b}$ has two distinct solutions $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, find two solutions of $A\boldsymbol{x} = \boldsymbol{0}$.

(b) Then find another (distinct) solution to $A\boldsymbol{x} = \boldsymbol{b}$.

P31. Find matrices A and B with the given property, or explain why you cannot:

(a) the only solution to $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is $\boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(b) the only solution to $B\boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is $\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

P32. Suppose $\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3$ are linearly-independent vectors in $\mathbb{R}^n$.

(a) Show that $\boldsymbol{v}_1 = \boldsymbol{w}_2 - \boldsymbol{w}_3$, $\boldsymbol{v}_2 = \boldsymbol{w}_1 - \boldsymbol{w}_3$, and $\boldsymbol{v}_3 = \boldsymbol{w}_1 - \boldsymbol{w}_2$ are linearly-*dependent*.

(b) Show that $\boldsymbol{z}_1 = \boldsymbol{w}_2 + \boldsymbol{w}_3$, $\boldsymbol{z}_2 = \boldsymbol{w}_1 + \boldsymbol{w}_3$, and $\boldsymbol{z}_3 = \boldsymbol{w}_1 + \boldsymbol{w}_2$ are linearly-*independent*.

P33. For each part, describe *all* cases where the rank is 2.

(a) For which numbers a and b does the matrix $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ have rank 2?

(b) For which numbers c and d does the matrix $B = \begin{bmatrix} 1 & 3 & 2 & 0 & 5 \\ 0 & 0 & c & 2 & 1 \\ 0 & 0 & 0 & d & 1 \end{bmatrix}$ have rank 2?

P34. Find a basis for, and the dimension of, these subspaces of 3×3 matrices:

(a) diagonal matrices

(b) symmetric matrices ($A^\top = A$)

(c) skew-symmetric matrices ($A^\top = -A$)

P35. Suppose $f_1(x), f_2(x), f_3(x)$ are three functions defined on the real line. The vector space they span could have dimension 1, 2, or 3. Give examples $\{f_1, f_2, f_3\}$ to show each possibility. (*Each example is a list of three functions, and an explanation.*)