Homework #3

Due Monday 31 January, 2022 at 11:59pm.

Submit as a single PDF by using Gradescope, via
canvas.alaska.edu/courses/7017
Problems from the textbook (Strang, *Intro Linear Algebra*, 5th ed. 2016) will be graded for completion, while the "**P**" Problems will be graded for correctness. Answers/solutions to Textbook Problems are linked at
bueler.github.io/math314/resources.html

Put these Textbook Problems first on your PDF, in this order.

from Problem Set 2.2, pages 53–57: # 1, 4, 6, 9, 11, 12, 14, 21, 27

from Problem Set 2.3, pages 66–69: # 1, 3, 4, 12, 19, 24, 28, 29

Put these "P" Problems next on your PDF, in this order.

P11. **(a)** Complete the right side to get a system which has no solutions:

$$\begin{aligned} 8x - 4y &= 14 \\ -2x + y &= \boxed{} \end{aligned}$$

(*There are many correct answers.*)

(b) Complete the right side to get a system which has infinitely-many solutions. (*There is only one correct answer.*)

(c) Write down two different solutions to the system in part **(b)**.

P12. Find three possible original problems (linear systems) so that elimination leads to $x - y = 1$ and $2y = -3$.

P13. Suppose we start with some 4 by 4 matrix A.

(a) E_{21} subtracts row 1 from row 2 and then P_{24} exchanges rows 2 and 4. What matrix $M = P_{24}E_{21}$ does both steps at once?

(b) P_{24} exchanges rows 2 and 4 and then E_{41} subtracts row 1 from row 4. What matrix $N = E_{41}P_{24}$ does both steps at once?

(c) Explain why $M = N$.

P14. Elimination on this 4 by 4 matrix Z will need matrices E_{21}, E_{32}, and E_{43}. What are these matrices?

$$Z = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

P15. Consider the matrices

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & b & 1 \end{bmatrix}.$$

(a) Compute AB and BA. Are they the same?

(b) Give formulas for A^n and B^n if $n \geq 1$ is an integer. (*Hint.* Compute A^2, A^3 and B^2, B^3. What is the pattern?)

P16. Consider this system $Ax = \boldsymbol{b}$:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 1 \\ 3 & 7 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$$

(a) Apply elimination to the augmented matrix $[A\ \boldsymbol{b}]$. How do you know this system has no solution?

(b) Change the last number 6 so that the new system does have a solution. Find a solution of the new system.