# Homework #12

## Due Monday 25 April, 2022 at 11:59pm.

Submit as a single PDF via Gradescope; see the Canvas page
canvas.alaska.edu/courses/7017
Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions to these Problems are linked at
bueler.github.io/math314/resources.html
The **P** Problems will be graded for correctness. When grading these Problems, I will expect you to write explanations using complete sentences!

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*Put these Textbook Problems first on your PDF, in this order.*

**from Problem Set 6.4, pages 344–348:** # 4, 5, 8

**from Problem Set 8.1, pages 406–409:** # 1, 3, 13, 17, 20, 24 (*Hint. Append columns.*)

**from Problem Set 8.2, pages 417–419:** # 1, 4, 5, 10, 11, 14, 27

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*Put these **P** Problems next on your PDF, in this order.*

**P57.** **(a)** By hand calculation, find an orthogonal matrix $Q$ which diagonalizes

$$S = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix}$$

(*Hints. Recall that if $X$ is an invertible matrix of eigenvectors of $A$ then $X$ diagonalizes $A$ in the sense that $AX = X\Lambda$ or $A = X\Lambda X^{-1}$, where $\Lambda$ is diagonal. Recall that the columns of an orthogonal matrix are orthonormal vectors: $Q^\top Q = I$. If $X = Q$ is orthogonal then $A = X\Lambda X^{-1} = Q\Lambda Q^\top$.*)

**(b)** Check your calculation using Matlab's `eig` command

```
[Q,D] = eig(S)
```

Explain any differences between your $Q$ and the computed `Q` from Matlab.

**P58.** **(a)** What matrix $A$ transforms $(1,0)$ and $(0,1)$ to $(r,s)$ and $(t,u)$?

**(b)** What matrix $B$ transforms $(a,b)$ and $(c,d)$ to $(1,0)$ and $(0,1)$?

**(c)** What condition on $a,b,c,d$ will make part **(b)** impossible?

**(d)** When $r = a$, $s = b$, $t = c$, and $u = d$ then $A$ and $B$ are matrix inverses. Confirm this.

**P59.** Consider the symmetric matrices

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad B = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

**(a)** Which of these classes of matrices do $A$ and $B$ belong to?:

INVERTIBLE, ORTHOGONAL, PROJECTION, PERMUTATION, DIAGONALIZABLE

Explain, or show work which supports your answers.

**(b)** Which of these factorizations are possible for $A$ and $B$?:

$LU$, $X\Lambda X^{-1}$, $Q\Lambda Q^\top$

(*As usual, $L$ is lower triangular with ones on diagonal, $U$ is upper triangular, $X$ is invertible, $\Lambda$ is diagonal, and $Q$ is orthogonal.*) Explain, or show work which supports your answers.

**(c)** By hand calculation, find $Q$ orthogonal and $\Lambda$ diagonal so that $B = Q\Lambda Q^\top$. (*Hint. $B$ has a repeated eigenvalue $\lambda = 0$, and you will need to find* two *orthogonal and normalized eigenvectors for this $\lambda$. Check your work in Matlab.*)

**P60.** In this problem we consider transformations from $\boldsymbol{V} = \mathbb{R}^2$ to $\boldsymbol{W} = \mathbb{R}^2$.

**(a)** For each of these transformations, is it linear? (*Show it is, or give a counterexample.*) In either case, give a simplified formula for $T(T(\boldsymbol{v}))$:

- $T(\boldsymbol{v}) = -\boldsymbol{v}$
- $T(\boldsymbol{v}) = \boldsymbol{v} + (1, 1)$
- $T(\boldsymbol{v}) =$ (do $90°$ rotation on $\boldsymbol{v}$) $= (-v_2, v_1)$
- $T(\boldsymbol{v}) =$ (projection) $= \frac{1}{2}(v_1 + v_2, v_1 + v_2)$

**(b)** Show that if $T$ is linear, i.e. $T(a\boldsymbol{v} + b\boldsymbol{w}) = aT(\boldsymbol{v}) + bT(\boldsymbol{w})$, then $T(T(\boldsymbol{v}))$ is also linear.

(*Note that it is common in mathematics to write "$T^2$" for the composition $T(T(\cdot))$ of a transformation $T$ with itself, even if $T$ is not linear, and/or $T$ is not already represented by a matrix.*)

**P61.** **(a)** Consider the vector space $\boldsymbol{M}$ of 2 by 2 matrices. Show that the transpose transformation $T$, on $\boldsymbol{M}$, is linear: $T(A) = A^\top$. (*Hint. Not much to do! Fits on one line.*)

**(b)** Try to find a 2 by 2 matrix $B$ so that $T(A) = BA$. Show that no such matrix $B$ exists! (*Hint. Show that $BA = A^\top$ being true for* all *matrices $A$ is impossible.*)

**(c)** If we rearrange the entries of a 2 by 2 matrix $A$ into a column vector $\boldsymbol{a}$ with four entries then we can do what is asked in **(b)**. That is, show that there is a $4 \times 4$ matrix $C$ so that $C\boldsymbol{a}$ is a column vector which is the rearrangement of $A^\top$.