Name:			

Math 314 Linear Algebra (Bueler)

Monday, 14 February 2022

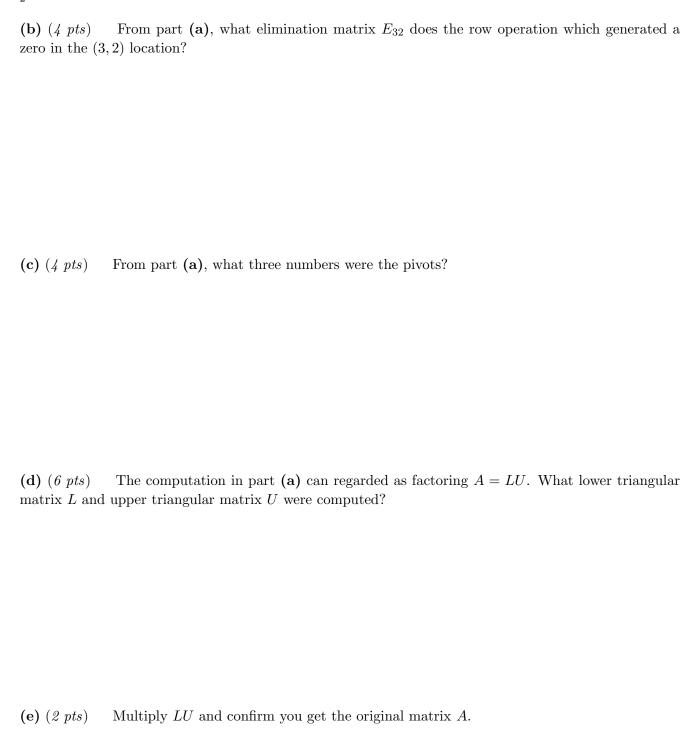
Midterm Exam 1

No book, notes, electronics, calculator, or internet access. 100 points possible. 65 minutes maximum.

1. Consider the following linear system $A\mathbf{x} = \mathbf{b}$:

$$2x_1 + x_2 = 6$$
$$-2x_1 + 3x_2 + 2x_3 = 0$$
$$2x_1 + 9x_2 + 9x_3 = 13$$

(a) (10 pts) Solve the linear system by *elimination* and then *back-substitution*. Use the standard algorithm. Show your work, and in particular show, as an intermediate stage, the triangular system which you get after elimination.



2. (10 pts) Suppose A is an invertible $n \times n$ matrix which has a known LU factorization into a lower triangular matrix L and an upper triangular matrix U. That is, suppose A = LU. Explain the steps, and name the algorithms, which you would use to solve a linear system $A\mathbf{x} = \mathbf{b}$. (Hint. The elimination stage has already been done. Don't propose to redo it!)

3. (8 pts) I have claimed that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Show (confirm) that this formula is correct. (Hint. A matrix multiplication suffices.)

- 4. True or false? Circle one. Give a short justification if true, and a counterexample if false.
- (a) (3 pts) A matrix with two equal columns is not invertible.

TRUE

FALSE

(b) (3 pts) Every triangular matrix with 1's down the main diagonal is invertible.

TRUE

FALSE

(c) (3 pts) If A is symmetric, so $a_{ij} = a_{ji}$, then A is invertible.

TRUE

FALSE

(d) (3 pts) If AB and BA are defined then A and B are square.

TRUE

FALSE

(e) (3 pts) If A and B are square matrices of the same size then $(A + B)^2 = A^2 + 2AB + B^2$.

TRUE

FALSE

5. (a) (10 pts) Invert this matrix by the Gauss-Jordan method:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Please show your work!

(b) (2 pts) What is the determinant of the matrix in part (a)?

6. (6 pts) Find the angle between these vectors. (Hint. You can write the answer in terms of an inverse trigonometric function, but otherwise everything should be simplified.)

$$\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

7. (6 pts) Sketch the column picture of this linear system. Noting that the solution values (x, y) play a particular role in this picture, find them and then show the solution in the sketch:

$$x - 2y = 0$$

$$x + y = 3$$

(Hint. Clearly sketch and label the 3 known vectors. Now, how to combine 2 of them to get the third?)

8. Consider the linear system

$$ax - 2y = 1$$

$$x + 4y = 3$$

(a) (4 pts) For which number a does elimination break down permanently, so that there are no solutions?

(b) (4 pts) For which number a does elimination break down temporarily, so that a row swap allows a solution?

(c) (3 pts) For the value of a found in part (b), solve the system.

Extra Credit. (3 pts) Find the quadratic polynomial $p(x) = a + bx + cx^2$ which passes through the points (-1,1), (1,5), (3,17).

9.
$$(6 \ pts)$$
 Suppose $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$. What is L^{-1} ?

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