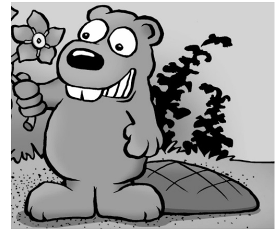
SOLVING DIFFERENTIAL EQUATIONS BY SERIES (§6.2)



Team effort! Find at least the first *five* coefficients $(c_0, c_1, c_2, c_3, c_4)$.

Find the solution of the ODE IVP by an appropriate power series:

$$y' + (x-1)y = 3, \quad y(1) = 2$$

this tells you the base point

$$y(x) = c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + \dots$$

$$y'(x) = c_1 + 2c_2(x-1) + 3c_3(x-1)^2 + 4c_4(x-1)^3 + \dots$$

$$\begin{aligned} y' + (x-1)y &= c_1 + 2c_2(x-1) + 3c_3(x-1)^2 + 4c_4(x-1)^3 + \dots \\ &\quad + c_0(x-1) + c_1(x-1)^2 + c_2(x-1)^3 + \dots \\ &= c_1 + (2c_2 + c_0)(x-1) + (3c_3 + c_1)(x-1)^2 \\ &\quad + (4c_4 + c_2)(x-1)^3 + \dots \\ &= 3 + 0(x-1) + 0(x-1)^2 + 0(x-1)^3 \end{aligned}$$

so:

$c_0 = y(1) = 2$) initial cond.

$c_1 = 3$

DE

$2c_2 + c_0 = 0 \longrightarrow c_2 = \frac{-c_0}{2} = -1$

$3c_3 + c_1 = 0 \longrightarrow c_3 = \frac{-c_1}{3} = \frac{-3}{3} = -1$

$4c_4 + c_2 = 0 \longrightarrow c_4 = \frac{-c_2}{4} = \frac{+1}{4}$

$5c_5 + c_3 = 0 \longrightarrow c_5 = \frac{-c_3}{5} = \frac{+1}{5}$

$\vdots$

$$\boxed{y(x) = 2 + 3(x-1) - (x-1)^2 - (x-1)^3 + \tfrac{1}{4}(x-1)^4 + \tfrac{1}{5}(x-1)^5 + \dots}$$

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Team effort! Find at least the first *five* coefficients (c_0, c_1, c_2, c_3, c_4).

Use the power series method to solve the initial value problem:

$$(x+1)y'' + y = 0, \quad y(0) = 0,\ y'(0) = 1$$

basepoint $a=0$

$$y(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots$$

$$y'(x) = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + \dots$$

$$y''(x) = 2c_2 + 3\cdot 2 c_3 x + 4\cdot 3 c_4 x^2 + 5\cdot 4 c_5 x^3 + \dots$$

$$\left.\begin{aligned} c_0 &= y(0) = 0 \\ c_1 &= y'(0) = 1 \end{aligned}\right\}$$ initial conds.

$$(x+1)y'' + y = 2c_2 x + 3\cdot 2 c_3 x^2 + 4\cdot 3 c_4 x^3 + 5\cdot 4 c_5 x^4 + \dots$$
$$+ 2c_2 + 3\cdot 2 c_3 x + 4\cdot 3 c_4 x^2 + 5\cdot 4 c_5 x^3 + \dots$$
$$+ 0 + x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots$$

$$= 2c_2 + (2c_2 + 3\cdot 2 c_3 + 1)x + (3\cdot 2 c_3 + 4\cdot 3 c_4 + c_2)x^2$$
$$+ (4\cdot 3 c_4 + 5\cdot 4 c_5 + c_3)x^3 + \dots$$

$$= 0 + 0x + 0x^2 + 0x^3 + 0x^4 + \dots$$

the DE says this

SO:

$$2c_2 = 0 \longrightarrow c_2 = 0$$

$$2c_2 + 3\cdot 2 c_3 + 1 = 0 \longrightarrow c_3 = -1/6$$

$$3\cdot 2 c_3 + 4\cdot 3 c_4 + c_2 = 0 \longrightarrow c_4 = -\frac{\cancel{3}\cdot 2 c_3}{4\cdot \cancel{3}} = \frac{1}{12}$$

$$\boxed{y(x) = x - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \dots}$$

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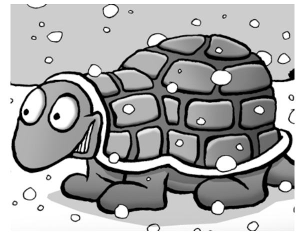

Team effort! Find at least the first *five* coefficients (c_0, c_1, c_2, c_3, c_4).

Use the power series method to solve the initial value problem:

$$y'' + (x+1)y = 0, \quad y(0) = 1,\ y'(0) = 0$$

basepoint $a=0$

$$y(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$$

$$y'(x) = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \cdots$$

$$y''(x) = 2c_2 + 3\cdot 2 c_3 x + 4\cdot 3 c_4 x^2 + \cdots$$

$$\left.\begin{aligned} c_0 &= y(0) = 1 \\ c_1 &= y'(0) = 0 \end{aligned}\right\} \text{ initial conds.}$$

$$\begin{aligned} y'' + (x+1)y &= 2c_2 + 3\cdot 2 c_3 x + 4\cdot 3 c_4 x^2 + 5\cdot 4 c_5 x^3 + \cdots \\ &\quad + c_0 x + c_1 x^2 + c_2 x^3 + c_3 x^4 + c_4 x^5 + \cdots \\ &\quad + c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots \\ &= 0 + 0x + 0x^2 + 0x^3 + \cdots \end{aligned}$$

DE says this

so:

$$2c_2 + c_0 = 0 \longrightarrow c_2 = \frac{-1}{2}$$

$$3\cdot 2 c_3 + c_0 + c_1 = 0 \longrightarrow c_3 = \frac{-1}{6}$$

$$4\cdot 3 c_4 + c_1 + c_2 = 0 \longrightarrow c_4 = \frac{+1/2}{4\cdot 3} = \frac{1}{24}$$

$$5\cdot 3 c_5 + c_2 + c_3 = 0 \longrightarrow c_5 = \frac{\frac{1}{2} + \frac{1}{6}}{5\cdot 3} = \frac{4}{6\cdot 5\cdot 3} = \frac{2}{45}$$

$$\vdots$$

$$\boxed{y(x) = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{2}{45}x^5 + \cdots}$$