3.1 (more) Linear Models

a lecture for MATH F302 Differential Equations

Ed Bueler, Dept. of Mathematics and Statistics, UAF

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for textbook: D. Zill, *A First Course in Differential Equations with Modeling Applications*, 11th ed.

linear models

- we got a good start on this material back in §1.3; it is not really new ...
- this section §3.1 requires ability to solve these types of DEs:
  1. the easiest DE: $y' = ky$
  2. separable linear equations from §2.2: $y' = g(x)y$ or $y' = ay + b$
  3. general linear equations from §2.3: $y' + P(x)y = f(x)$
- §2.3 method handles all first-order *linear* equations (3), but other methods are usually quicker for forms 1 or 2 , if you have a choice

- these slides simply contain 5 exercises from §3.1 in this order:
  # 4, 42, 17, 36, 37

exercise 4

4. *The population of bacteria in a culture grows at a rate proportional to the number of bacteria present at time t. After 3 hours it is observed that 400 bacteria are present. After 10 hours 2000 bacteria are present. What was the initial number of bacteria?*

exercise 42

42. Fluctuating Population *The differential equation $dP/dt = (k \cos t)P$, where k is a positive constant, is a mathematical model for a population $P(t)$ that undergoes yearly seasonal fluctuations. Solve the equation subject to $P(0) = P_0$. Use a graphing utility to graph the solution for different choices of P_0 [and k].*

exercise 42, cont.

exercise 17

17. *A thermometer reading* $70^\circ F$ *is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads* $110^\circ F$ *after* $\frac{1}{2}$ *minute and* $145^\circ F$ *after 1 minute. How hot is the oven?*

exercise 17, cont.

36. How High?—No Air Resistance *Suppose a small cannonball weighing 16 pounds is shot vertically upward, as shown in the Figure, with an initial velocity* $v_0 = 300 ft/s$*. The answer to "How high does the cannonball go?" depends on if we take air resistance into account.*

(a) *Suppose air resistance is ignored. If the positive direction is upward then a model for the height* $s(t)$ *of the cannonball is given by* $m\, d^2s/dt^2 = -mg$ *or equivalently* $d^2s/dt^2 = -g$*. Since* $ds/dt = v(t)$ *the last differential equation is the same as* $dv/dt = -g$*. We take* $g = 32 ft/s^2$*. Find the velocity* $v(t)$ *of the cannonball at time* t*.*

exercise 36, cont.

(b) *Use the result in part* **(a)** *to determine the height* $s(t)$ *of the cannonball, measured from ground level. Find the maximum height attained by the cannonball.*

37. How High?—Linear Air Resistance *Repeat exercise 36, but this time assume that air resistance is proportional to instantaneous velocity. In particular, suppose that the constant of proportionality is* $k = 0.25$.[1] *It stands to reason that the maximum height must be less than computed in exercise 36. Show this.*

[1] Value for k changed to make effect more obvious.

exercise 37, cont.

exercise 37, cont. cont.

expectations

- to learn this material, just listening to a lecture is *not* enough!
- also:
  - *read* section 3.1 in the textbook
    - for quizzes and exams you must be able to handle the examples like 1-6 in this section
  - look for "modeling with differential equations" on YouTube or Google
  - *do* Homework 3.1