Math 302 Differential Equations: Quiz 5 **15 November 2023**

Name: ______________________________ | / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work; use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [5 points] Find the radius and interval of convergence of the power series $\sum_{k=1}^{\infty} \frac{(3x-1)^k}{k}$

$R =$

$I =$

2. [4 points] What is the (minimum) radius of convergence of the power series solution of the following differential equation, about the ordinary point $x_0 = 0$? Explain briefly.

(*Hint. Do not find the series solution itself!*)

$$(x^2 - 25)y'' + y' + xy = 0$$

$R =$

3. [8 points] Verify by direct substitution that the given power series is a solution of the differential equation.

$$y = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n!}, \qquad y' + 2y = 0$$

4. [8 points] Solve the initial value problem below by starting with a power series

$$y(x) = c_0 + c_1 x + c_2 x^2 + \cdots = \sum_{n=0}^{\infty} c_n x^n$$

(*Hints. You can use summation notation or not. Find at least the first five coefficients. You can check your answer by non-series methods, but full credit requires a valid series calculation.*)

$$y'' + 4y = 0, \qquad y(0) = 3, \quad y'(0) = 0$$

Extra Credit. [2 points] The energy function associated to a conservative physical system is $E(x,x') = \frac{m}{2}(x')^2 + P(x)$ where $P(x)$ is the potential energy. The negative derivative of the potential energy is the force: $P'(x) = -F(x)$. What is the energy for the undamped, nonlinear mass-spring system with equation $mx'' = -\dfrac{kx}{1+x^2}$? (*Assume* m, k *are positive constants.*)

$E(x,x') =$

Maclaurin Series	Interval of Convergence
$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$	$(-\infty, \infty)$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$	$(-\infty, \infty)$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$	$(-\infty, \infty)$
$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}x^{2n+1}$	$[-1, 1]$ (2)
$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{(2n)!}x^{2n}$	$(-\infty, \infty)$
$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}x^{2n+1}$	$(-\infty, \infty)$
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}x^n$	$(-1, 1]$
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$	$(-1, 1)$