

Name: _____

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25 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[9 points]** The rate at which an object cools depends on its exposed surface area S , among other things. We can include this effect in Newton's law of cooling:

$$\frac{dT}{dt} = kS(T_m - T).$$

Here k , S , and T_m are all constant, and k and S are positive.

- a) Find the general solution $T(t)$.

- b) Consider water bottles with water at initial temperature 10°C which are put outside when it is winter in Fairbanks, say at -20°C . Use this information to give an updated formula $T(t)$. (*Hint. Your formula will still have constants k and S unknown.*)

2. [4 points] Compute and simplify the following integral: $\int \frac{dz}{z(a-bz)}$

3. [4 points] Determine whether the following set of functions is linearly-independent on the interval $(-\infty, \infty)$:

$$f_1(x) = 5, \quad f_2(x) = \cos^2 x, \quad f_3(x) = \sin^2 x$$

Math 302 Differential Equations: Quiz 3

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4. [8 points] Consider this differential equation, and these functions:

$$y'' - y' - 12y = 0; \quad e^{-3x}, \quad e^{4x}$$

a) Verify that the given functions are solutions of the differential equation.

b) Show that they form a fundamental set of solutions.

c) Form the general solution.

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Extra Credit. [1 point] Returning to your answer to problem **1 b)** on the first page, show that the time it takes to reach the freezing point for the water in the bottles (0°C) is inversely proportional to the surface area of the bottle, all other things being equal.

EXTRA SPACE