Name: $\qquad$
25 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [4 points] The direction field of the following differential equation is shown: $\frac{d y}{d x}=1-x y$. For each of the following points, sketch an approximate solution curve.
a) $y(0)=0$
b) $y(2)=2$

2. [4 points] The same differential equation as in the previous question is linear: $\frac{d y}{d x}=1-x y$. Find the general solution. You may write the solution in integral form if you do not know how to do an integral.
3. [4 points] Find the general solution by separation of variables:

$$
\frac{d y}{d x}=e^{3 x+2 y}
$$

4. [5 points] Determine whether this differential equation is exact. If it is exact, solve it:
$x \frac{d y}{d x}=2 x e^{x}-y+6 x^{2}$
5. [8 points] For each of the following initial value problems, determine if the differential equation is separable, linear, or exact. Then find the solution.
a) $\quad x \frac{d y}{d x}+y=4 x+1, \quad y(1)=8$
b) $\quad \frac{d P}{d t}=-P^{2}, \quad P(0)=3$

Extra Credit. [1 point] The following differential equation (DE) is separable: $\frac{d y}{d x}=y^{2}-4$.
Suppose that we want to solve the initial value problem for this DE with $y(3)=2$. Explain very briefly why the separable (separation of variables) technique breaks down immediately. Then write down the solution anyway, without any calculation.

