Name: ———————————————————————   / 25

25 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[4 points]** Find values of m so that the function $y = e^{mx}$ is a solution of this differential equation:

$$y'' + 5y' + 6y = 0$$

2. **[6 points]**

a) Verify that $y = -1/(x+c)$ is a one-parameter family of solutions of the differential equation $y' = y^2$.

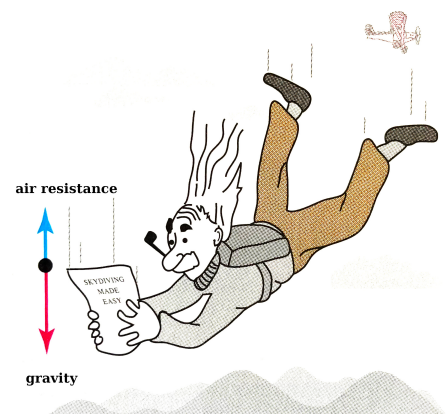*b)* Find the solution $y(x)$ of the initial value problem: $y' = y^2, \quad y(1) = 1.$

c) What is the largest interval of definition I for the solution in part *b)*?

3. [5 points] Verify that

$$y(x) = e^{-x^2} + e^{-x^2} \int_0^x e^{t^2}\, dt$$

is a solution to the differential equation $\dfrac{dy}{dx} + 2xy = 1$.

4. [4 points] Determine a differential equation for the (instantaneous) velocity $v(t)$ of a falling body of mass m if air resistance is proportional to the velocity. Assume the upward direction is positive. (*Hints. $ma = F$ where F is the net force. The only forces are the two shown in the figure. Denote the acceleration of gravity by $g > 0$.*)

5. [6 points] I claim that $x = c_1 \cos t + c_2 \sin t$ is a two-parameter family of solutions of the second-order differential equation $x'' + x = 0$.

a) Verify this claim.

b) Find the solution of the initial value problem with $x(\pi/2) = 0$ and $x'(\pi/2) = 1$.

Extra Credit. [1 point] Write down a differential equation which does not have any real solutions. Provide a one-sentence explanation of why it has no solutions. (*Hint. Do not be limited by normal form $dy/dx = f(x,y)$.*)

EXTRA SPACE