Name: SAMPLE

MATH F302 Differential Equations (Bueler)

SAMPLE Midterm Exam

No textbook or notes or calculator.
Please write your final answer in the box (if present).

1. (*10 pts*) Find the solution to the initial value problem:

$$\frac{dy}{dx} + 3y = x, \qquad y(0) = 5.$$

$y(x) = \boxed{}$

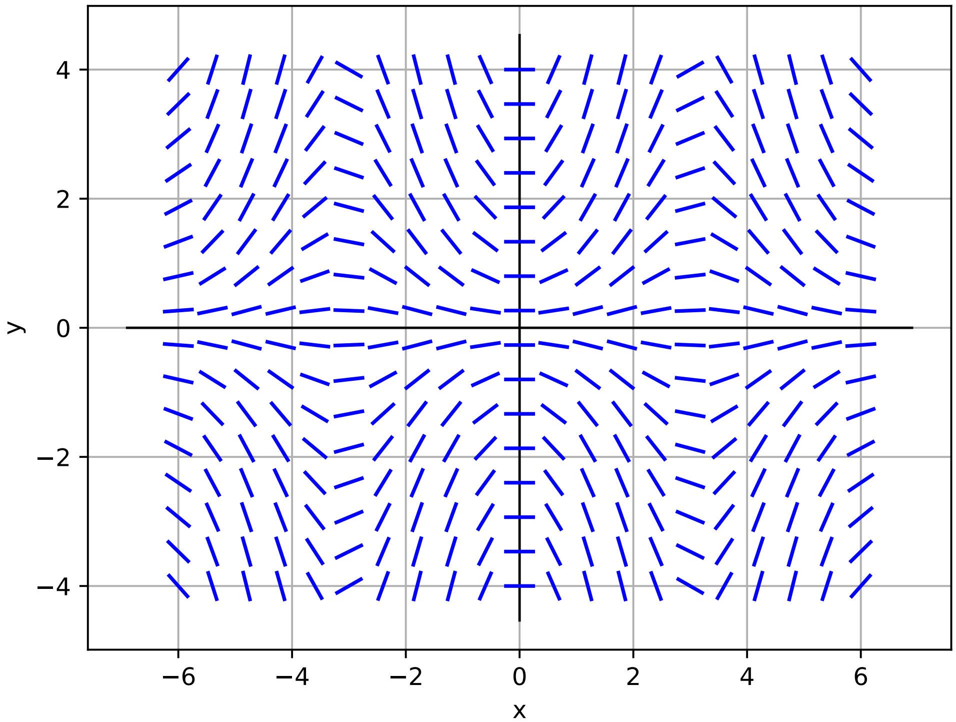
2. (*10 pts*) Verify that the family of functions is a solution of the given differential equation:

$$\frac{dP}{dt} = P(1 - P), \qquad P = \frac{c_1 e^t}{1 + c_1 e^t}$$

3. **(a)** (*5 pts*) The ODE

$$\frac{dy}{dx} = y \sin x$$

has the direction field shown below. Sketch the solution which passes through $(x_0, y_0) = (4, -2)$. Please extend the sketched solution to the interval $-6 \leq x \leq 6$.



(b) (*10 pts*) Solve the following initial value problem—*give an exact formula $y(x)$ for the solution!*. Then sketch the solution you find on the direction field above:

$$\frac{dy}{dx} = y \sin x, \qquad y(0) = 2.$$

$y(x) =$ $\boxed{}$

4. (*10 pts*) Find the general solution of the ODE:

$$\frac{dz}{dt} = e^{3t}e^{2z}$$

$z(t) =$ $\boxed{}$

5. **(a)** (*10 pts*) Show that the following differential equation is exact:

$$(5y - 2x)y' - 2y = 0$$

(b) (*10 pts*) Find the general solution to the differential equation in part **(a)**.

6. **(a)** (*10 pts*) A drug is infused into a patient's bloodstream, and we denote the amount (grams) of the drug by $x(t)$. The infusion is at a constant rate of r grams per second. Simultaneously, the drug is removed by the biochemical process in the patient's body at a rate proportional to the amount $x(t)$ of drug which is present at time t. Using k for the constant of proportionality, determine a differential equation (DE) for the amount $x(t)$.

(b) (*10 pts*) Assume there is no drug in the patient's bloodstream at time $t = 0$. Solve this initial value problem, which includes the DE in part **(a)**. (*Your solution will contain constants r and k.*)

$$x(t) = \boxed{}$$

Extra Credit. (*3 pts*) A more realistic version of problem 6 would include the fact that a drug infusion does not go on forever. Assume it stops after time $T > 0$. Write down an ODE IVP for this improved model, and sketch a representative solution. (*The equation is solvable in pieces, but an exact solution is not needed here.*)

7. Consider the initial value problem: $y'' + y = 0, \quad y(0) = 2,\ y'(0) = 0$

($\mathit{10\ pts}$) Find the solution. Show your steps.

$y(x) =$ $\boxed{}$

8. (*10 pts*) Find the general solution: $y^{(4)} - 3y'' - 18y = 0$

(*Hint: Certain 4th-order polynomials can be factored if you know how to factor quadratics.*)

$y(x) =$ $\boxed{}$