Name: _____

Midterm Exam

In-class. No book, notes, electronics, calculator, internet access, or communication with other people. Please write your solution in the box if provided. 100 points possible. 65 minutes maximum!

1. (*6 pts*) Solve the differential equation by separation of variables: $\dfrac{dy}{dx} - 2xy^2 = 0$

$y(x) =$

2. (*6 pts*) Verify that $y(t) = 1/(c - t)$ is a one-parameter family of solutions of the differential equation $\dfrac{dy}{dt} = y^2$.

3. (*15 pts*) Determine whether the differential equation is exact. If it is exact, solve it and write the solution in the box: $\left(8y^3 - 4x\right)\dfrac{dy}{dx} = 5x + 4y$

4. (*15 pts*) Solve the initial value problem, assuming L, R, E, i_0 are constants:

$$L\frac{di}{dt} + Ri = E, \quad i(0) = i_0$$

$i(t) =$

5. **(a)** (*8 pts*) Find the general solution: $y'' - y' - 12y = 0$

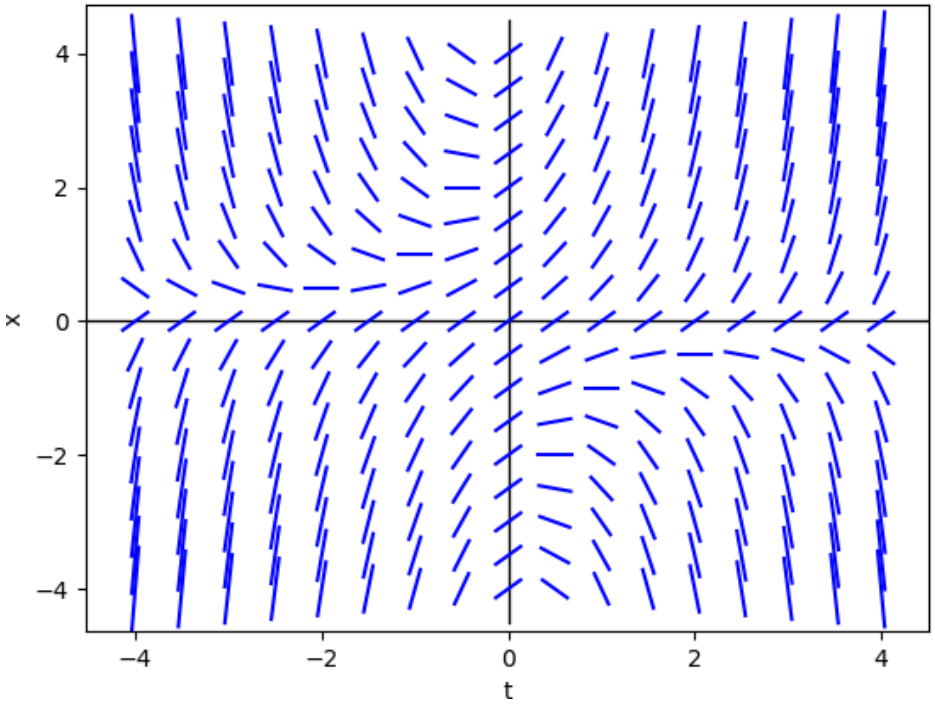
$y(x) =$

(b) (*8 pts*) Show that your general solution in part **(a)** is built from a fundamental set of solutions.

6. **(a)** (*5 pts*) The direction field of the following differential equation is shown: $\dfrac{dx}{dt} = 1 + tx$

For each of the following points, sketch an approximate solution curve.

- $x(0) = -1$
- $x(-2) = 2$



(b) (*8 pts*) Find the general solution of the differential equation in part **(a)**. You may write the solution in integral form if you do not know how to do an integral.

$x(t) =$

7. **(a)** (*5 pts*) Newton's law of cooling/warming says that the rate of change of an object's temperature is proportional to the difference between its temperature and a constant ambient temperature. Write this differential equation, a model for the temperature $T(t)$. (*Hint. This model has two constants.*)

(b) (*5 pts*) Find the general solution of the differential equation in part **(a)**.

$T(t) =$

(c) (*5 pts*) A thermometer—*it is the object here*—is initially at room temperature 20° C. It is put in boiling water at 100° C. After 2 seconds the thermometer reads 60° C. What is the constant of proportionality in the above model?

8. **(a)** (*7 pts*) Find the general solution of this third-order, constant-coefficient, homogeneous, and linear differential equation: $y''' + y' = 0$

$y(x) =$

(b) (*7 pts*) The general solution in part **(a)** is a linear combination of three specific functions $f_1(x), f_2(x), f_3(x)$. State these functions and then compute and simplify the Wronskian.

$W(f_1, f_2, f_3) =$

Extra Credit. (*3 pts*) Find the general solution on $(0, \infty)$: $xy'' - y' = 0$

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