

WORKSHEET: EXERCISES ON H^1 NORMS AND THE POISSON EQUATION

1. Let $f(x) = \sin(3x)$.

(a) Compute the $H^1(-\pi, \pi)$ norm of f via the original definition.

(b) Compute the $H^1(S^1) = H_{per}^1(-\pi, \pi)$ norm of f via the Fourier series (summability-of-coefficients) definition.

2. Let $f(x, y) = 6y(x - x^2) + 2(y - y^3)$ be the data on the unit square $\Omega = (0, 1)^2$.

(a) Show that $u(x, y) = (x - x^2)(y - y^3)$ solves the (strong form) of the homogeneous Dirichlet problem for the Poisson equation on Ω : $-\nabla^2 u = f$, $u = 0$ on $\partial\Omega$.

(b) Describe in a few words, if you can, how I must have come up with this example.

2

3. Let $u(x, y) = \sin(\pi x)e^{\pi y}$ on the unit square $\Omega = (0, 1)^2$.

(a) Compute the H^1 norm of u .

(b) Show that u solves the (strong form) of the Laplace equation on Ω .

(c) Describe a boundary value problem, on Ω , which u satisfies.