Assignment 2

Due Friday 6 February at the beginning of class ⟵ *revised twice!*

This Assignment is based on Chapters 1 and 2 of our textbook,[1] and on the lectures.

DO THE FOLLOWING Exercises from Chapter 2 (see pages 29–31; 2.1.6 removed):

- **Exercise 2.1.1** *Note that Saxe defines* open *and* closed *sets on page 16. Please use those definitions.*
- **Exercise 2.1.4**
- **Exercise 2.1.7** *State your claim, and then prove it.*
- **Exercise 2.1.10**
- **Exercise 2.1.13** *Do parts **(a)** and **(b)** only.*
- **Exercise 2.2.2** *You may use, without proof, the fact that a countable union of countable or finite sets is countable.*
- **Exercise 2.3.1**
- **Exercise 2.3.5** *To show that the formula defines a norm, you may use, without proof, the triangle inequality for integrals, that is,* $|\int_E f(t)\,dt| \le \int_E |f(t)|\,dt$.
- **Exercise 2.3.8** (Extra Credit) *You will use the fact that* $\mathbb{R}$ *is complete.*

DO THE FOLLOWING ADDITIONAL PROBLEMS.

P3. *In lecture*[2] *I solved a classic boundary value problem (BVP) by writing* $u(x)$ *as an integral of the data* $\phi(x)$*. This is nearly the same, or very slightly easier. In fact* $\phi \in L^1(0,1)$ *suffices, but we need not worry about generality for now.*

Consider the following BVP, with data $\phi \in C([0,1])$:

$$\begin{aligned} -u''(x) &= \phi(x) \quad \text{for all } x \in (0,1) \\ u(0) &= 0 \\ u'(1) &= 0 \end{aligned}$$

Solve this problem by integration. In particular, write the solution as

$$u(x) = \int_0^1 k(x,t)\phi(t)\,dt,$$

and give the precise formula for $k(x,t)$. Is k a continuous function of its two variables? Is it bounded?

[1] K. Saxe, *Beginning Functional Analysis*, Springer 2010.

[2] See these slides: https://bueler.github.io/fa/assets/slides/S26/week2.pdf.