Final Exam: Prove 3 Nontrivial Theorems

Friday 1 May 2026, 10:15am–12:15pm, Chapman 107

For the in-class Final Exam you will **prove 3 nontrivial theorems** which we have seen during the semester. A list of 7 possibilities appears below, and these are the only possibilities. You will have this document in your hand during the Exam, but otherwise you must remember what you want to say! **You may NOT bring notes, the textbook, or electronics of any kind to the Exam.** Just bring a writing implement.

It is essential that you **draft and practice both the theorem statements and the proofs**. Make sure to read relevant sections of the textbook[1] or slides[2]. Get feedback on your drafts from other students or friends (or me). Decide in advance how you will remember enough detail so as to recreate the theorem statement and its proof during the Exam itself.

This is a writing assignment. I will grade for correctness and completeness (the 2 highest priorities), and then for readability.

Instructions:

For each theorem, use a separate sheet. Identify it by "**T**x" in the upper-left corner, and put your name in the upper-right. Clearly and precisely state the theorem, making sure that both the hypotheses and conclusion are correct. Then write "Proof." at the beginning, and end the proof with "$\square$". Full credit will only be given for a well-written, appropriately-detailed, and correct proof. (Starting your proof with a brief statement of its strategy is a good idea because partial credit can be given even if details are incomplete or incorrect.) Each theorem should use at most 2 pages.

T1. Theorem 2.7, page 23: $(C([a,b]), \|\cdot\|_\infty)$ is complete.

T2. Theorem 3.15, page 53: Lebesgue's dominated convergence theorem.

T3. Theorem 3.19, page 61: For $1 \le p < \infty$, $(L^p(\mu), \|\cdot\|_p)$ is a normed vector space. (*You may assume Hölder's inequality, but you need to prove the triangle inequality.*)

T4. Theorem 4.2, page 82: Bessel's inequality.

T5. Theorem 5.4, page 98: If X, Y are normed vector spaces, and if Y is complete, then $\mathcal{B}(X,Y)$ is complete. (*You may assume that the operator norm is a norm.*)

T6. Riesz-Fréchet representation theorem: If $(X, \langle\cdot,\cdot\rangle)$ is a Hilbert space and if $\ell \in X'$ then there is a unique representer $x_\ell \in X$ so that $\ell(y) = \langle y, x_\ell\rangle$ for all $y \in X$. (*See zoom recording for Wed. 1 April, at time-stamp about 12:50. You may assume the Hilbert decomposition theorem.*)

T7. The Riesz representation proof that the weak-form Poisson problem with homogeneous boundary conditions is well-posed. (*See the week 13 slides, slides 21–23.*)

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[1] K. Saxe, *Beginning Functional Analysis*, Springer 2010.

[2] On the Daily Log tab at `bueler.github.io/fa/daily.html`.