Name:

Math 617 Functional Analysis (Bueler)

Friday 3 May 2024

Final Quiz

In-class or proctored. No book, notes, electronics, calculator, internet access, or communication with other people. <u>Precise</u> statements of definitions, theorems, and lemmas are expected. 100 points possible. <u>65 minutes</u> maximum.

1. (5 *pts*) Define a **complex Hilbert space**.

2. $(5 \ pts)$ Define a **unitary map**.

3. (10 pts) For an operator T on a complex Hilbert space, whether bounded or unbounded, define the spectrum $\sigma(T)$ and the resolvent set $\rho(T)$.

4. $(15 \ pts)$ State the **Riesz lemma**, which describes the dual space of a (complex) Hilbert space.

5. (10 pts) For a (generally) unbounded operator T on a complex Hilbert space, define the domain $\mathcal{D}(T^*)$ of the adjoint, and the action of the adjoint T^* .

6. (10 pts) For a (generally) unbounded operator on a complex Hilbert space, defined what it means to be **symmetric**, and what it means to be **self-adjoint**.

7. (15 pts) State the Riesz (or Riesz-Markov-Kakutani) representation theorem.

8. $(15 \ pts)$ State the multiplication-operator form of the spectral theorem for self-adjoint operators.

9. (15 pts) State the **bounded Borel functions** form of the **functional calculus for** self-adjoint operators.

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