SOLUTIONS

Math 253 Calculus III (Bueler) 17 April 2023 *Not turned in!*

Worksheet: Computing potentials

1. Is the 2D vector field $\mathbf{F} = \langle \sin y, x\cos y\rangle$ conservative? If it is, compute a potential $f(x,y)$ so that $\mathbf{F} = \nabla f$.

$P_y = \cos y$, $Q_x = \cos y$ ✓ $\therefore$ conservative $\Rightarrow$ $f_x = \sin y$

$f(x,y) = x\sin y + g(y)$

$x\cos y = f_y = x\cos y + g'(y) \Rightarrow g'(y) = 0$, $g(y) = c.$

$f(x,y) = x\sin y$

with or without "+c" is fine

2. Is the 3D vector field $\mathbf{F} = \langle 2x, e^z, ye^z - 1\rangle$ conservative? If it is, compute a potential $f(x,y,z)$ so that $\mathbf{F} = \nabla f$.

$P = 2x$, $Q = e^z$, $R = ye^z - 1$

$P_y = 0$, $P_z = 0$, $Q_x = 0$, $Q_z = e^z$, $R_x = 0$, $R_y = e^z$ $\therefore$ conservative $\Rightarrow$ $f_x = 2x$

$f(x,y,z) = x^2 + g(y,z)$

$e^z = f_y = 0 + g_y$

$g(y,z) = ye^z + h(z)$

$f(x,y,z) = x^2 + ye^z + h(z)$

$ye^z - 1 = f_z = 0 + ye^z + h'(z)$

$h'(z) = -1$

$h(z) = -z \rightarrow f(x,y,z) = x^2 + ye^z - z$

3. Is the 3D vector field $\mathbf{F} = \langle y\sin(x), e^z, y\rangle$ conservative? If it is, compute a potential $f(x,y,z)$ so that $\mathbf{F} = \nabla f$.

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ y\sin x & e^z & y \end{vmatrix} = \underbrace{(1 - e^z)}_{\neq 0}\hat{\imath} - (0 - 0)\hat{\jmath} + (0 - \sin x)\hat{k}$$

so $\rightarrow$ $R_y \neq Q_z$

so: not conservative

$$\int 2xye^{x^2y}\,dx \underset{\leftarrow u=x^2y}{=} \int e^u\,du = e^u + C = e^{x^2y}+C$$

4. Is the 2D vector field $\mathbf{F} = (2xye^{x^2y})\mathbf{i} + (x^2e^{x^2y})\mathbf{j}$ conservative? If it is, compute a potential $f(x,y)$ so that $\mathbf{F} = \nabla f$.

$$P = 2xye^{x^2y} \Rightarrow P_y = 2xe^{x^2y} + 2xye^{x^2y}x^2$$
$$Q = x^2e^{x^2y} \Rightarrow Q_x = 2xe^{x^2y} + x^2e^{x^2y}2xy$$

yes conservative

$$f_x = P = 2xye^{x^2y}$$
$$f(x,y) = e^{x^2y} + h(y)$$
$$x^2e^{x^2y} = f_y = e^{x^2y}x^2 + h'(y)$$
$$h'(y) = 0 \Rightarrow h(y) = c$$

$$f(x,y) = e^{x^2y}$$

5. Is the 3D vector field $\mathbf{F} = (2xy)\mathbf{i} + (x^2 + 2yz)\mathbf{j} + y^2\mathbf{k}$ conservative? If it is, compute a potential $f(x,y,z)$ so that $\mathbf{F} = \nabla f$.

$$\nabla \times \vec{F} = (2y-2y)\hat{i} - (0-0)\hat{j} + (2x-2x)\hat{k} = \vec{0}$$

yes conservative

$$f_x = 2xy$$
$$f(x,y,z) = x^2y + g(y,z)$$
$$x^2+2yz = f_y = x^2 + g_y(y,z)$$
$$g_y(y,z) = 2yz$$
$$g(y,z) = y^2z + h(z)$$

$$f(x,y,z) = x^2y + y^2z + h(z)$$
$$y^2 = f_z = 0 + y^2 + h'(z)$$
$$0 = h'(z)$$
$$C = h(z)$$

$$f(x,y,z) = x^2y + y^2z$$

EXTRA SPACE