SOLUTIONS

Worksheet: Double integrals over rectangles

1. Evaluate the double (iterated) integral:

$$\int_0^\pi \int_0^{\pi/2} \cos x \sin(3y)\, dx\, dy = \int_0^\pi \left(\int_0^{\pi/2} \cos x\, dx \right) \sin(3y)\, dy$$

$$= \int_0^\pi \left(\sin x \Big]_0^{\pi/2} \right) \sin 3y\, dy$$

$$= 1 \cdot \int_0^\pi \sin 3y\, dy = -\frac{1}{3}\cos(3y)\Big]_0^\pi = -\frac{1}{3}(-1-1) = \frac{2}{3}$$

2. Evaluate the double (iterated) integral:

$$\int_1^2 \int_3^4 x^5 + y^6\, dy\, dx = \int_1^2 \left(\int_3^4 x^5 + y^6\, dy \right) dx$$

$$= \int_1^2 \left[x^5 y + \frac{y^7}{7} \right]_3^4 dx = \int_1^2 4x^5 + \frac{4^7}{7} - 3x^5 - \frac{3^7}{7}\, dx$$

$$= \frac{x^6}{6} + \frac{4^7-3^7}{7} x \Big]_1^2 = \frac{2^6}{6} + \frac{4^7-3^7}{7}\cdot 2 - \frac{1}{6} - \frac{4^7-3^7}{7}\cdot 1$$

$$= \boxed{\frac{2^6-1}{6} + \frac{4^7-3^7}{7}}$$

3. Write the integral as an iterated integral in the two different ways:

$$\iint_R e^{\cos(xy)} x^2\, dA \qquad \text{where } R = [0,1]\times[-1,1]$$

$$= \int_{-1}^1 \int_0^1 e^{\cos(xy)} x^2\, dx\, dy$$

or

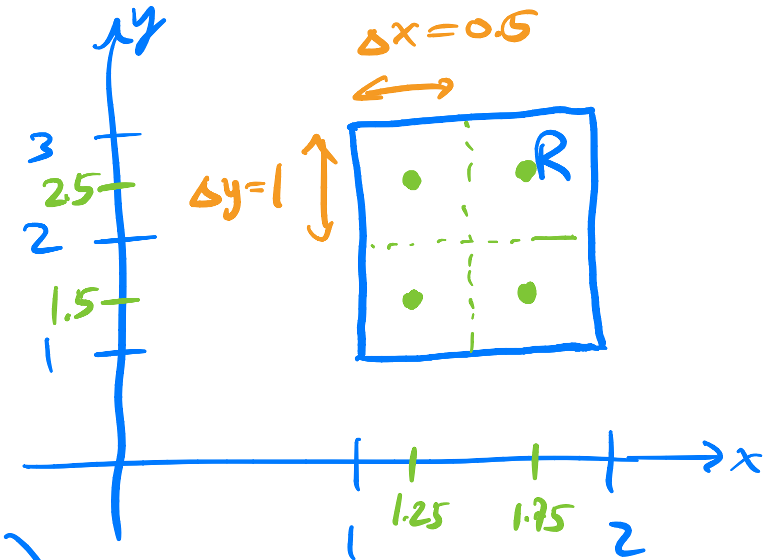
$$= \int_0^1 \int_{-1}^1 e^{\cos(xy)} x^2\, dy\, dx$$

I don't know how to do either of these by hand. And it was not requested.

4. Apply the midpoint rule to estimate the integral. Use $m = 2$ points in the x-direction and $n = 2$ points in the y-direction

$$\iint_R \frac{1}{xy}\,dA \qquad \text{where } R = [1,2]\times[1,3]$$

$$\approx \Delta x \Delta y \big(f(1.25, 1.5) + f(1.75, 1.5) + f(1.25, 2.5) + f(1.75, 2.5) \big)$$



$$= 0.5\left(\frac{1}{(1.25)(1.5)} + \frac{1}{(1.75)(1.5)} + \frac{1}{(1.25)(2.5)} + \frac{1}{(1.75)(2.5)} \right)$$

$$= 0.7314$$

I used $m=100$, $n=200$ in Matlab to get $\iint_R \frac{1}{xy}\,dA \approx 0.761494$

versus $(\ln 3)(\ln 2) = 0.761500$ from below.

5. Compute the integral in problem **4** exactly. How close is the midpoint rule estimate?

$$\int_1^2 \int_1^3 \frac{1}{xy}\,dy\,dx = \int_1^2 \left(\int_1^3 \frac{1}{xy}\,dy \right) dx$$

$$= \int_1^2 \frac{1}{x}\left[\ln y\right]_1^3 dx = \ln 3 \int_1^2 \frac{1}{x}\,dx$$

$$= \ln 3\left[\ln x\right]_1^2 = (\ln 3)(\ln 2) = 0.7615$$