SOLUTIONS

Worksheet: Limits of functions of 2 or 3 variables

1. Compute the limit: $\displaystyle\lim_{(x,y)\to(2,3)} \frac{1}{y} - \frac{4}{x} = \frac{1}{3} - \frac{4}{2} = \frac{1}{3} - 2 = -\frac{5}{3}$

easy because $f(x,y) = \frac{1}{y} - \frac{4}{x}$ is continuous at $(2,3)$

2. Compute the limit: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} + \sqrt{y}} \overset{0/0}{=} \lim_{(x,y)\to(0,0)} \frac{x(x-y)}{\sqrt{x}+\sqrt{y}}$

$$= \lim_{(x,y)\to(0,0)} \frac{x\left((\sqrt{x})^2 - (\sqrt{y})^2\right)}{\sqrt{x}+\sqrt{y}} = \lim_{(x,y)\to(0,0)} \frac{x\,\cancel{(\sqrt{x}+\sqrt{y})}(\sqrt{x}-\sqrt{y})}{\cancel{\sqrt{x}+\sqrt{y}}}$$

$$= \lim_{(x,y)\to(0,0)} x(\sqrt{x} - \sqrt{y}) = 0(0-0) = 0$$

easy: fcn is continuous

3. Where is the function $f(x,y) = \dfrac{1}{y(x-1)}$ continuous? Find and sketch the largest region.

$f(x,y)$ is continuous where it is defined:

$\{(x,y) \mid y \neq 0 \text{ and } x \neq 1\}$

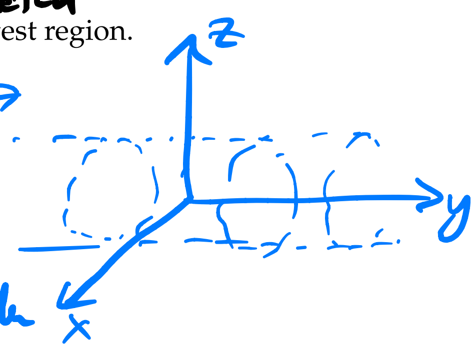
4. Where is the function $g(x,y,z) = \dfrac{1}{x^2+z^2-1}$ continuous? Find and sketch the largest region.

$g(x,y,z)$ is continuous where it is defined:

$\{(x,y,z) \mid x^2+z^2 \neq 1\}$

note: $x^2+z^2=1$ is a cylinder

continuous except on cylinder



5. If $f(x,y) = x^2 - 3y$, find the limit: $\displaystyle\lim_{h\to0} \frac{f(1+h,y) - f(1,y)}{h} =$

$$= \lim_{h\to0} \frac{(1+h)^2 - 3y - [1-3y]}{h} = \lim_{h\to0} \frac{\cancel{1} + 2h + h^2 \cancel{-3y} \cancel{-1} \cancel{+3y}}{h}$$

$$= \lim_{h\to0} \frac{\cancel{h}(2+h)}{\cancel{h}} = \lim_{h\to0} 2+h = 2+0 = 2$$