SOLUTIONS

Worksheet: Just calculate some cross products!

The memorable formula for calculating cross products uses a determinant:

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}\mathbf{k}$$

Here is the hard-to-remember "simplified" formula:

$$\mathbf{u}\times\mathbf{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1\rangle$$

In problems **A–D**, *compute* $\mathbf{u}\times\mathbf{v}$. *Then follow the instructions in problems* **E–F**.

A. $\mathbf{u} = \langle 1,0,0\rangle$, $\mathbf{v} = \langle 0,0,1\rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}\hat{\imath} - \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}\hat{k} = 0\hat{\imath} - \hat{\jmath} + 0\hat{k}$$

$$= -\hat{\jmath} = \langle 0,-1,0\rangle$$

[compare right-hand rule: $\hat{\imath}\times\hat{k} = -\hat{\jmath}$]

B. $\mathbf{u} = \langle 1,2,3\rangle$, $\mathbf{v} = \langle 4,5,6\rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = (2\cdot 6 - 3\cdot 5)\hat{\imath} - (1\cdot 6 - 3\cdot 4)\hat{\jmath} + (1\cdot 5 - 2\cdot 4)\hat{k}$$

$$= \langle -3, 6, -3\rangle = -3\hat{\imath} + 6\hat{\jmath} - 3\hat{k}$$

C. $\mathbf{u} = x\mathbf{i} - y\mathbf{j}$, $\mathbf{v} = z\mathbf{j} - w\mathbf{k}$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & -y & 0 \\ 0 & z & -w \end{vmatrix} = ((-y)(-w) - 0)\hat{\imath} - (x(-w) - 0)\hat{\jmath} + (x\cdot z - 0)\hat{k}$$

$$= yw\hat{\imath} + xw\hat{\jmath} + xz\hat{k} = \langle yw, xw, xz\rangle$$

D. $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, $\mathbf{u} = \mathbf{v}$

$$\vec{u}\times\vec{v} = \vec{u}\times\vec{u} = \vec{0}$$

[and: $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a & b & c \\ a & b & c \end{vmatrix} = (bc - bc)\hat{\imath} - (ac - ac)\hat{\jmath} + (ab - ba)\hat{k}$
$= 0\hat{\imath} - 0\hat{\jmath} + 0\hat{k}$]

E. Find the *triple scalar product* $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ where $\mathbf{u} = \langle 1, 1, 1 \rangle$, $\mathbf{v} = \langle 7, 6, 9 \rangle$, and $\mathbf{w} = \langle 4, 2, 7 \rangle$. (*Hint. Compute* $\mathbf{v} \times \mathbf{w}$ *first.*)

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 6 & 9 \\ 4 & 2 & 7 \end{vmatrix} = (42-18)\hat{i} - (49-36)\hat{j} + (14-24)\hat{k}$$

$$= 24\hat{i} - 13\hat{j} - 10\hat{k}$$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = 1 \cdot 24 + 1 \cdot (-13) + 1 \cdot (-10)$$

$$= 24 - 23 = 1$$

F. Compute the ordinary determinant $\begin{vmatrix} 1 & 1 & 1 \\ 7 & 6 & 9 \\ 4 & 2 & 7 \end{vmatrix}$.

$$= 1 \cdot (6 \cdot 7 - 2 \cdot 9) - 1 \cdot (7 \cdot 7 - 4 \cdot 9) + 1 \cdot (7 \cdot 2 - 4 \cdot 6)$$

$$= (42 - 18) - (49 - 36) + (14 - 24)$$

$$= 24 - 13 - 10 = 1$$

general rule:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

so E & F are the same

Note you can check your work in Matlab.

For example, I checked B and F:

```
>> cross([1 2 3],[4 5 6])
ans
     -3     6    -3
>> det([1 1 1; 7 6 9; 4 2 7])
ans     1
```

spaces