

Worksheet: Just calculate some cross products!

The memorable formula for calculating cross products uses a determinant:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \mathbf{k}$$

Here is the hard-to-remember "simplified" formula:

$$\mathbf{u} \times \mathbf{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

In problems **A–D**, compute $\mathbf{u} \times \mathbf{v}$. Then follow the instructions in problems **E–F**.

A. $\mathbf{u} = \langle 1, 0, 0 \rangle, \mathbf{v} = \langle 0, 0, 1 \rangle$

B. $\mathbf{u} = \langle 1, 2, 3 \rangle, \mathbf{v} = \langle 4, 5, 6 \rangle$

C. $\mathbf{u} = x\mathbf{i} - y\mathbf{j}, \mathbf{v} = z\mathbf{j} - w\mathbf{k}$

D. $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}, \mathbf{u} = \mathbf{v}$

E. Find the *triple scalar product* $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ where $\mathbf{u} = \langle 1, 1, 1 \rangle$, $\mathbf{v} = \langle 7, 6, 9 \rangle$, and $\mathbf{w} = \langle 4, 2, 7 \rangle$.
(Hint. Compute $\mathbf{v} \times \mathbf{w}$ first.)

F. Compute the ordinary determinant $\begin{vmatrix} 1 & 1 & 1 \\ 7 & 6 & 9 \\ 4 & 2 & 7 \end{vmatrix}$.