

Name: \_\_\_\_\_

/ 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[4 points]** Sketch the vector field  $\mathbf{F}(x, y) = \mathbf{i} - \mathbf{j}$ . Describe the vector field in a sentence. (*Draw at least 6 vectors in your sketch. Regarding the sentence, what is the length of the vectors, and their directions?*)

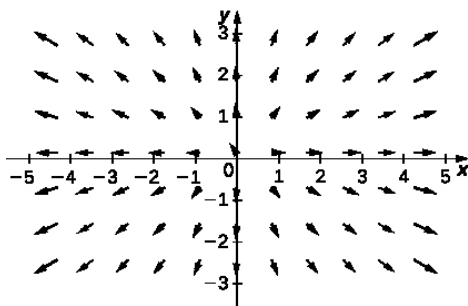
2. **[3 points]** Compute the gradient vector field of  $f(x, y, z) = xy - yz + z^3$ .

3. [6 points] Find the work done by the force field  $\mathbf{F}(x, y) = -2y\mathbf{i} + 2x\mathbf{j} + (x + y)\mathbf{k}$  in moving an object along the curve  $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} - 3\mathbf{k}$ , where  $0 \leq t \leq \pi$ .

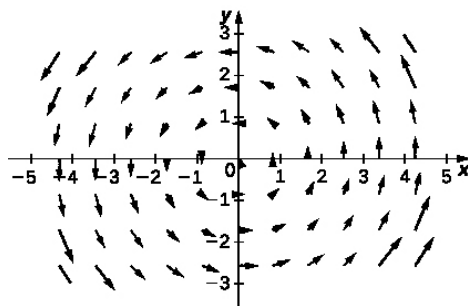
4. [3 points] Find  $\mathbf{T}(t)$ , the unit tangent vector field, for the curve  $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ .

5. [6 points] Evaluate the line integral  $\int_C f(x,y) ds$  of the scalar function  $f(x,y) = x + y$  along the straight-line path connecting the origin to the point  $(1, 1)$ .

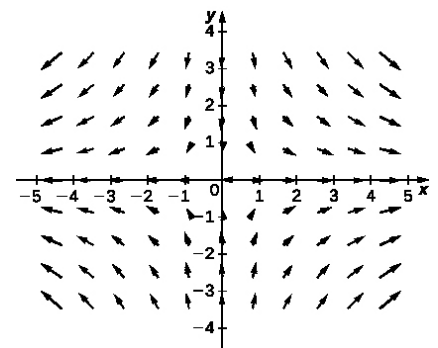
6. [3 points] Consider the vector fields  $\mathbf{F} = -y\mathbf{i} + x\mathbf{j}$ ,  $\mathbf{G} = x\mathbf{i} + y\mathbf{j}$ , and  $\mathbf{H} = x\mathbf{i} - y\mathbf{j}$ . Match  $\mathbf{F}$ ,  $\mathbf{G}$ ,  $\mathbf{H}$  to their graphs below. In particular, write the variable name below the matching graph.



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**Extra Credit. [1 point]** I claim  $\mathbf{F}(x, y) = \langle e^x, y + e^x \rangle$  is a conservative vector field. Find a potential  $f(x, y)$ .

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EXTRA SPACE FOR ANSWERS