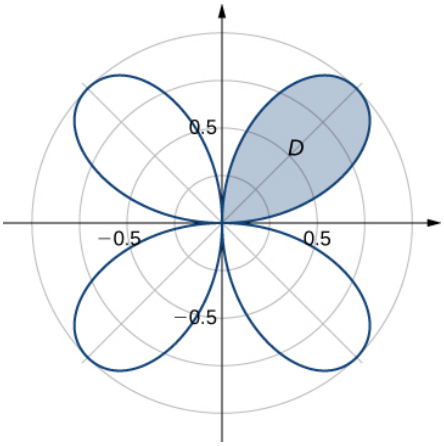
Math 253: Quiz 8 **Thursday 30 March, 2023**

Name: _____ **/ 25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [5 points] Find the area of one leaf of the rose $r = \sin(2\theta)$, which is shown in the figure.



2. [5 points] Convert the integral to polar coordinates. There is no need to evaluate the integral! (**Hint.** Sketch the region of integration, which tells you the limits on the r, θ integrals.)

$$\int_0^4 \int_{-\sqrt{16-x^2}}^{+\sqrt{16-x^2}} \arctan(x^2+y^2)\, dy\, dx =$$

3. [5 points] Using mathematically-correct steps, show that:

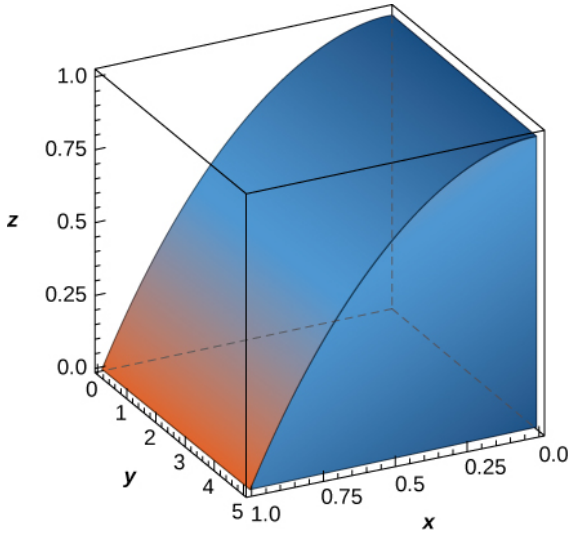
$$\int_a^b \int_c^d \int_e^f F'(x)G'(y)H'(z)\,dz\,dy\,dx = [F(b)-F(a)]\,[G(d)-G(c)]\,[H(f)-H(e)]$$

(**Hint.** Start on the left. What terms can be moved out of the inner integrals? What do you know about the integral of a derivative?)

4. [5 points] Assume $B = \left\{(x,y,z) \,\middle|\, 1 \le x \le 2, 0 \le y \le 2, 1 \le z \le 3\right\}$. Evaluate the triple integral:

$$\iiint_B xy\,dV =$$

5. [5 points] A solid object is shown. It is the set in the first octant which bounded by $z = 1 - x^2$ and the plane $y = 5$. Supposing its density is $\rho(x,y,z) = 1 + x + y$, **completely set up** a triple integral to find its total mass.



EXTRA SPACE FOR ANSWERS

Extra Credit. [1 point] Compute and fully simplify the integral in problem **5**.