

Name: _____

/ 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [5 points] Evaluate the iterated (double) integral:

$$\int_1^2 \int_0^9 \frac{\sqrt{y}}{x^2} dy dx =$$

2. [5 points] Set up, but do not evaluate, a double integral to compute the **area** of the region between the graph $x = \frac{1}{2}y^2$ and the graph $y = \tan x$, over the interval $0 \leq y \leq 1$. (**Hint.** Carefully sketch the region.)

3. [8 points] Consider the surface $z = x^2 + y^2$ and the region $R = \{(x, y) \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}$.

a) Sketch the surface.

b) Sketch the region R .

c) Find the volume under the surface and above the region R .

4. [7 points] Suppose we have a plate of metal in the shape of a quarter circle, bounded by the curves $x = 0$, $y = 0$, and $x^2 + y^2 = 4$. Find the average temperature of the plate if the temperature is given by the function $T(x, y) = 10x$.

Extra Credit. [1 point] Suppose $f(x)$ is any continuous function and consider the domain

$$D = \{(x, y) \mid a \leq x \leq b, f(x) \leq y \leq f(x) + 1\}.$$

Use a double integral to show that the area of D is just $A_D = b - a$.

EXTRA SPACE FOR ANSWERS