

Name: _____

/ 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [7 points] Consider the function $f(x, y) = e^x \cos y$.

a) Compute the gradient $\nabla f(x, y)$.

b) Compute the directional derivative of f at the point $P\left(1, \frac{\pi}{2}\right)$ in the direction $\mathbf{v} = -\mathbf{i}$.

2. [5 points] Find the maximum rate of change of $f(x,y) = x \ln y$ at the point $(2,1)$, and the direction in which it occurs.

3. [5 points] Sketch the level curve of $f(x,y) = 3x^2 + 3y^2$ which passes through the point $P(1,1)$, and draw the gradient vector at P .

4. [8 points] Consider the function $f(x, y) = x^3 + y^3 - 3x - 12y - 2$.

a) Find all the critical points.

b) For each critical point, use the second derivative test to determine if it is a local minimum, local maximum, or saddle point.

Extra Credit. [1 point] Show that the gradient of a function $f(x,y)$ is orthogonal to its level curves. (**Hint.** Write down the equation for a level curve. Suppose the level curve is parameterized. Take derivatives of both sides of the equation.)

EXTRA SPACE FOR ANSWERS