Math 253: Quiz 5 SOLUTIONS **2 March, 2023**

Name: _____

/ 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [5 points] Find the tangent plane to the surface $f(x,y) = 9x^2 - y^3$ at the point $P(1,2,1)$. and simplify

$f_x = 18x, \quad f_y = -3y^2$

$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$= 1 + 18\cdot 1\,(x-1) + (-3\cdot 2^2)(y-2)$

$z = 1 + 18(x-1) - 12(y-2)$

either is fine

or: $18x - 12y - z + 7 = 0$

2. [5 points] Let $w(t,v) = \sin(tv)$ where $t = r+s$ and $v = rs$. Find $\frac{\partial w}{\partial s}$.

$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial t}\frac{\partial t}{\partial s} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial s}$

$= \cos(tv)\cdot v \cdot 1 + \cos(tv)\cdot t \cdot r$

$= (v + tr)\cos(tv)$

either is fine

$= (rs + (r+s)r)\cos((r+s)rs)$

3. [8 points] The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$.

a) Find the differential dV.

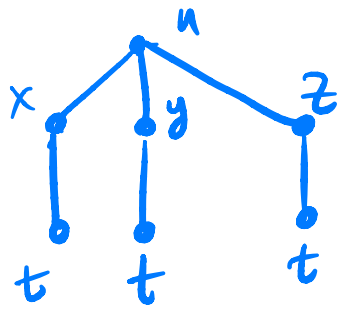
$$dV = \frac{2}{3}\pi r h\, dr + \frac{1}{3}\pi r^2\, dh$$

b) A machine makes cones for ice cream, with target values $r = 3$ cm and $h = 10$ cm, thus a target volume of $V = 30\pi\,\text{cm}^3$. However, the machine is only accurate to within 1 cm in r and h. Use the differential to estimate the maximum deviation in volume away from the target volume.

$$dV = \frac{2}{3}\pi \cdot 3 \cdot 10 \cdot \underbrace{1}_{dr} + \frac{1}{3}\pi 3^2 \cdot \underbrace{1}_{dh}$$

$$= 20\pi + 3\pi = 23\pi\ \text{cm}^3$$

4. [4 points] Let $u = u(x,y,z)$ where $x = x(t), y = y(t), z = z(t)$. For $\frac{du}{dt}$, show a tree diagram and state the chain rule.



$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

5. [3 points] What is a normal vector to the plane $36x + 6y + z - 39 = 0$?

$$\vec{n} = \langle 36, 6, 1 \rangle$$

Extra Credit. [1 point] The first-order Taylor polynomial of $f(x)$ at the basepoint $x = a$ is

$$p_1(x) = f(a) + f'(a)(x-a).$$

What is the first-order Taylor polynomial of $f(x,y)$ at the basepoint $(x,y) = (a,b)$? Use correct notation.

$p_1(x,y) =$ $f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

[I pointed out in class that the linearization, or tangent plane, is the Taylor polynomial of degree 1.]

EXTRA SPACE FOR ANSWERS