Math 253: Quiz 5 **2 March, 2023**

Name: _____ **/ 25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[5 points]** Find and simplify the tangent plane to the surface $f(x,y) = 9x^2 - y^3$ at the point $P(1,2,1)$.

2. **[5 points]** Let $w(t,v) = \sin(tv)$ where $t = r+s$ and $v = rs$. Find $\dfrac{\partial w}{\partial s}$.

3. [8 points] The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$.

a) Find the differential dV.

b) A machine makes cones for ice cream, with target values $r = 3$ cm and $h = 10$ cm, thus a target volume of $V = 30\pi\,\text{cm}^3$. However, the machine is only accurate to within 1 cm in r and h. Use the differential to estimate the maximum deviation in volume away from the target volume.

4. **[4 points]** Let $u = u(x,y,z)$ where $x = x(t), y = y(t), z = z(t)$. For $\dfrac{du}{dt}$, show a tree diagram and state the chain rule.

5. **[3 points]** What is a normal vector to the plane $36x + 6y + z - 39 = 0$?

Extra Credit. [1 point] The first-order Taylor polynomial of $f(x)$ at the basepoint $x = a$ is

$$p_1(x) = f(a) + f'(a)(x-a).$$

What is the first-order Taylor polynomial of $f(x,y)$ at the basepoint $(x,y) = (a,b)$? Use correct notation.

$p_1(x,y) =$

EXTRA SPACE FOR ANSWERS